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Optimal surrender of guaranteed minimum maturity benefits under stochastic volatility and interest rates

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ABSTRACT

In this paper we analyse how the policyholders' surrender behaviour is influenced by changes in various sources of risk impacting a variable annuity (VA) contract embedded with a guaranteed minimum maturity benefit rider that can be surrendered anytime prior to maturity. We model the underlying mutual fund dynamics by combining a Heston (1993) stochastic volatility model together with a Hull and White (1990) stochastic interest rate process. The model is able to capture the smile/skew often observed on equity option markets (Grzelak and Oosterlee 2011) as well as the influence of the interest rates on the early surrender decisions as noted from our analysis. The annuity provider charges management fees which are proportional to the level of the mutual fund as a way of funding the VA contract. To determine the optimal surrender decisions, we present the problem as a 4-dimensional free-boundary partial differential equation (PDE) which is then solved efficiently by the method of lines (MOL) approach. The MOL algorithm facilitates simultaneous computation of the prices, fair management fees, optimal surrender boundaries and hedge ratios of the variable annuity contract as part of the solution at no additional computational cost. A comprehensive analysis on the impact of various risk factors in influencing the policyholder's surrender behaviour is carried out, highlighting the significance of both stochastic volatility and interest rate parameters in influencing the policyholder's surrender behaviour. With the aid of the hedge ratios obtained from the MOL, we construct an effective dynamic hedging strategy to mitigate the provider's risk and compare different hedging performances when the policyholders' surrender behaviour is either optimal or sub-optimal.

1. Introduction

Variable annuities (VAs) are long-dated contracts which are now dominating the market for retirement income products in most developed countries such as US, Japan and across Europe. As of June 2015, the variable annuity net assets in the US alone were in excess of $1.9 trillion, surpassing pre-Global financial crisis peaks of $1.5 trillion (Holland and Simonelli, 2015). A variable annuity is a binding contract between an annuity provider and policyholder where the policyholder agrees to pay a fixed premium either as a single payment or a stream of periodic payments during the accumulation phase. In return, the annuity provider undertakes to make guaranteed minimum periodic payments starting either immediately or at a deferred future date.

Variable annuities provide policyholders the flexibility to participate in the equity market while returning minimum guarantee levels in the event of poor performance of the underlying mutual fund. There are two major categories of guarantees embedded in VAs, namely guaranteed minimum death benefits (GMDBs) and guaranteed minimum living benefits (GMLBs) (see Bauer et al., 2008; Ignatieva et al., 2016). A GMDB is usually offered during the accumulation phase and it provides a guaranteed sum to beneficiaries in the event of untimely death of the policyholder. GMLBs offer living protection to the policyholder's income against market risk by guaranteeing a variety of benefits which can be classified as the GMxB, where "x" stands for maturity (M), income (I) and withdrawal (W). A GMMB guarantees the return of the premium payments made by the policyholder or a higher stepped-up value at the end of the accumulation period. A GMIB guarantees an income stream over an agreed period of time when the policyholder purchases a retirement annuity or annuitizes a GMMB regardless of the underlying investment performance. A GMWB guarantees the policyholder a stream of withdrawals cumulatively summing

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to the initial investment throughout the life of the contract conditional on the policyholder being alive.

Guarantees embedded in variable annuity contracts are usually funded by proportional fees levied from the underlying mutual fund. This paper aims to provide insights on the risks associated with trading a variable annuity contract embedded with a GMMB rider by taking the perspective of a rational policyholder who can optimally surrender the contract anytime prior to maturity.[1] Bernard et al. (2014) note that if the guarantee is deep-out-of-the-money, it may be optimal for the policyholder to surrender the contract prior to maturity as a way of avoiding paying high fees. The authors formulate the valuation problem using the geometric Brownian motion (GBM) framework and then use numerical integration techniques to analyse optimal surrender regions from the perspective of the policyholder. Such surrender behaviour poses significant hazard to annuity providers' solvency, hence it is imperative to properly analyse the embedded options in VA contracts (Grosen and Jorgensen, 2000). As a way of discouraging policyholders from surrendering early, annuity providers normally charge penalty fees which takes a variety of functional forms. Bernard et al. (2014) and Shen et al. (2016) incorporate a penalty fee structure which is exponentially decreasing with time to maturity. Other penalty fee structures are discussed in Milevsky and Salisbury (2001) who denote such fees as deferred surrender charges.

Shen and Xu (2005) consider the valuation of equity-linked policies with interest rate guarantees in the presence of surrender options using the partial differential equation approach under the GBM environment. A similar problem is presented in Constabile et al. (2008) who devise a binomial tree approach to determine fair premium values. Bacinello (2013) also values participating life insurance policies with surrender options using a recursive binomial tree approach. Shen et al. (2016) take the annuity provider's perspective and use numerical quadrature techniques to derive expressions for fair management fees and the associated optimal surrender boundaries using the framework developed in Bernard et al. (2014).

The majority of the literature mentioned above has been premised under the GBM framework. Given the long-term nature of variable annuity contracts, it is crucial to accurately quantify all the major risk factors impacting the underlying fund dynamics (Coleman et al., 2006; Du and Martin, 2014; Kling et al., 2014). Contrary to the log-normal asset return distribution assumptions under the GBM framework (Black and Scholes, 1973), significant empirical studies have revealed that such distributions exhibit leptokurtic features and are characterised by heavy tails (Platen and Rendek, 2008). Empirical evidence also suggests that volatility of asset returns is not constant (see Christoffersen et al. (2009), Jang et al. (2014) among others). In this regard, van Haastrecht et al. (2010) highlight the importance of stochastic volatility when pricing guaranteed annuity options; contracts equivalent to GMMBs with an additional feature of converting accumulated funds into a life annuity. Kang and Meyer (2014) also note that the level of volatility of the interest rates plays a crucial role in influencing the exercise decisions of American style options prior to maturity (equivalent to surrender decisions under the current context).

Shah and Bertsimas (2010) use Monte Carlo simulation to assess the impact of both stochastic volatility and interest rates on guaranteed lifelong withdrawal benefits by making comparison with the GBM framework. The authors note that the valuations vary substantially depending on the modelling framework used. Kélani and Quittard-Pinon (2017) develop a unified valuation framework for pricing and hedging various GMLBs under the Lévy market and note that traditional modelling assumption of using the GBM framework undervalues economic capital required by providers to hedge such guarantees.

There has been less focus on the development of a realistic modelling framework for analysing the impact of various sources of risk in influencing the surrender behaviour. Such an analysis is critical to all players in the variable annuity business as it can be used as key reference when making risk management decisions. In filling this gap, the aims of this paper are twofold; the first aim involves taking the policyholder's perspective by presenting a comprehensive analysis on how the surrender behaviour is influenced by the interaction of various risk factors impacting a VA contract embedded with a GMMB rider. In so doing, we extend the framework presented in Bernard et al. (2014) by incorporating both stochastic volatility and stochastic interest rate in our valuation framework. We assume that the policyholder's premium is invested in an underlying mutual fund which evolves under the influence of stochastic volatility (Heston, 1993) and stochastic interest rates (Hull and White, 1990).

For the second aim we take the variable annuity provider's perspective in devising a dynamic hedging algorithm for immunising the provider's net liability anytime prior to maturity of the contract. We extend the framework presented in Bernard and Kwak (2016) who consider a GMMB rider that can only be exercised at maturity when the underlying fund dynamics evolves according to the geometric Brownian motion process. There has been increasing focus on hedging of variable annuity contracts; Coleman et al. (2007) use local risk minimising strategies for hedging GMDB riders. Alonso-Garcia et al. (2017) devise a Fourier cosine based approach for pricing and hedging GMWB riders embedded in variable annuity contracts. For hedging, Alonso-Garcia et al. (2017) develop strategies that seek to minimise moment and quantile-based risk measures, such as the variance of the hedging outcomes or the 95% VaR of the hedged portfolio loss distribution. To aid our numerical analysis in this paper, we utilise the method of lines (MOL) technique (Kang and Meyer, 2014) as a tool for generating fair management fees, early surrender profiles and hedge ratios which are important ingredients for risk management.

The remainder of the paper is structured as follows: Section 2 presents the modelling framework and formulates the corresponding value function as a free-boundary problem. Section 3 outlines the MOL approach for solving the free-boundary problem. This method generates, as part of the solution, optimal surrender profiles and the associated hedge ratios which can be used in the construction of appropriate hedging strategies. A dynamic hedging framework is presented to hedge the provider's risk in Section 4. Section 5 contains all numerical results analysing how various sources of risk influence surrender decisions and hedging performance when the policyholders surrender either optimally or sub-optimally. Concluding remarks are contained in Section 6.

2. Problem statement

As highlighted above, we consider how the policyholder behaviour is influenced by various sources of risk impacting a VA contract embedded with a GMMB rider for the case where the contract can be surrendered anytime prior to maturity subject

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[1] In reality, policyholders tend to sub-optimally surrender contracts, with such decisions driven by various factors which include changes to the policyholder's financial and personal circumstances (see Bauer et al. (2015) for a detailed discussion of the underlying drivers of policyholder exercise behaviour).