



# Robust evaluation of SCR for participating life insurances under Solvency II

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## ABSTRACT

This article proposes a robust framework to evaluate the solvency capital requirement (SCR) of a participating life insurance with death benefits. The preference for robustness arises from the ambiguity caused by the market incompleteness, model shortcomings and parameters misspecifications. To incorporate the uncertainty in the procedure of evaluation, we consider a set of potential equivalent pricing measures in the neighborhood of the real one. In this framework, closed form expressions for the net asset value (NAV) and for its moments are found. The SCR is next approximated by the Value at Risk of Gaussian or normal inverse Gaussian (NIG) random variables, approaching the NAV distribution and fitted by moments matching.

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## 1. Introduction

In the Solvency II regulation (first pillar), the Solvency Capital Requirement (SCR) is meant to cover one year of deterioration of the Net Asset Value (NAV). The NAV, that is a market consistent evaluation of future profits yield by the company. It is evaluated by the difference between the market value of assets and the Best Estimate (BE) provisions. The BE is appraised by a market to model approach, as the expected sum of future discounted benefits. Whereas, the total market value of liabilities is the sum of this BE and of the Risk Margin (RM).

However, the Solvency II framework presents some operational drawbacks. Firstly, due to the complexity of guidelines, SCR and NAV are evaluated exclusively by Monte Carlo simulations in most of insurance companies. As underlined in [Floryszczak et al. \(2016\)](#), programs computing the SCR are black boxes, extremely demanding in terms of resources and not adapted for decision making. For this reason, the regulator introduced the second Pillar, called “Own Risk Solvency Assessment” (ORSA) which ensures that the management has a holistic view of risks. In the ORSA, the capital assessment may substantially differ from the first pillar, and the insurer has the freedom to develop alternative approaches to manage risks. Furthermore, the undertaking should ensure that its assessment of the overall solvency needs is forward-looking,

including a medium term or long term perspective as appropriate. Within this context, [Bonnin et al. \(2014\)](#) and [Combes et al. \(2016\)](#) propose analytical models to pilot the asset–liability management (ALM) policy of participating policies. This article proposes a new alternative to these approaches.

The Solvency II regulation also raises interesting theoretical questions. Firstly, a single insurance claim cannot be hedged on an individual basis. Instead, insurers rely on the law of large numbers and pool risks to reduce their exposure to claims. Due to this incompleteness, multiple equivalent pricing measures exist and each one reflects the insurer's risk aversion for unhedgeable risks. Solvency II recommends to evaluate BE provisions under a risk neutral measure with realistic assumptions about unhedgeable risks. But there are no guidelines to determine admissible measures. A second point concerns the model risk. The SCR is evaluated by complex programs and today the potential impact of model misspecifications on the SCR is not addressed by Solvency II. This article is a first attempt to incorporate this uncertainty in the SCR valuation.

However, the uncertainty about the risk neutral measure, parameters and the model gives rise to a current of research in the literature that focuses on robustness. A model is qualified as robust if it takes into account the potential misspecifications. The theory of Robustness was pioneered in economics by [Hansen and Sargent \(1995\)](#), (2001) or (2001). This theory is an alternative to Bayesian approaches that are typically limited to parametric versions of model uncertainty. In a robust approach, there is no need to make any assumptions about the a priori distribution of parameters and the uncertainty concerns the entire drift function.

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To formulate model misspecifications, Hansen and Sargent employ a relative entropy factor. This relative entropy captures the perturbation between the estimated model and the unobservable true model. Anderson et al. (2003) extend the robust control theory with the theory of semi-groups. Balter and Pelsser (2015) use a similar approach and propose a robust pricing method in an incomplete market. Maenhout (2004) proposes a robust solution to the consumption and portfolio problem of Merton (1969). Instead of explicitly bounding the entropy, a penalty term is introduced in the infinitesimal generator of the value function. This additional term penalizes alternative models that are too far away from the reference model. This approach was extended in Maenhout (2006) to mean reverting risk premiums. The literature on robustness is vast and we refer to Guidolin and Rinaldi (2010) for a detailed review.

This paper contributes to the literature in two main directions. Firstly, it proposes a robust and simple model to estimate the SCR of participating life insurances with death benefits. The evaluation scheme is mainly based on analytical expressions of NAV and BE, without any recourse to simulations. Our approach is also compliant with ORSA guidelines and provides a simple method to estimate prospective SCR. Secondly, this article addresses the uncertainty surrounding the model specifications. The distance between the estimated model and the unobservable true specification is delimited by an entropic constraint on eligible real and pricing measures. The bound on the entropy is directly related to the level of confidence in results produced by the model. The ambiguity around model assumptions may then be counterbalanced by adjusting this entropic bound. If we position our work in the scope of the ORSA, the constraint on entropy may be calibrated so as to match BE and SCR estimates computed by our model with those obtained with a complex internal model. Our approach may also be reconciled with the rationale of Cochrane and Saa Requejo's (2000) Good-Deal-Bound. Their idea is to bound the Sharpe ratios of all possible assets in the market and thus exclude Sharpe ratios which are considered to be too large. The idea was streamlined and extended to models with jumps in Bjork and Slinko (2006) or models with switching regimes as in Donnelly (2011).

This article is structured in the following way. Section 2 defines the multivariate Brownian motion driving the financial market in which the insurance company invested collected premiums. The model for the human mortality is presented in Section 3. The following section introduces the specifications of the participating insurance contract with death benefits. In Section 5, we discuss the choice of the pricing measure and of the entropic constraint. Closed form expressions for robust and non-robust best estimate provisions are next provided. In Section 7, we infer closed form expressions for the net asset value (NAV) and for its moments. In Section 8, the SCR is calculated by a Value at Risk of Gaussian or normal inverse Gaussian (NIG) random variables, approaching the NAV distribution and fitted by moments matching. Section 9 discusses the problem of ambiguity under the real measure.

## 2. The financial market

We consider an insurance company that proposes participating life contracts with a minimal guarantee, and death benefits. Before detailing the specifications of policies, we firstly introduce the financial market. We consider  $d$  assets driven by a multivariate geometric Brownian motion of dimension  $d$ . This process is defined on a probability space  $\Omega$ , endowed with a filtration  $(\mathcal{F}_t)_t$  under a real probability measure denoted by  $P$ . There is considerable piece of evidence suggesting that the Brownian motion with constant drift and standard deviation are not appropriate to model stocks returns, due to extreme comovements. However, it is analytically tractable and its shortcomings are compensated in Section 5 by integrating preferences for robustness. The assets prices are denoted

by  $S_t^i$  for  $i = 0$  to  $d - 1$  and obey to following stochastic differential equations:

$$\underbrace{\begin{pmatrix} \frac{dS_t^0}{S_t^0} \\ \frac{dS_t^1}{S_t^1} \\ \vdots \\ \frac{dS_t^{d-1}}{S_t^{d-1}} \end{pmatrix}}_{\frac{dS_t^S}{S_t^S}} = \underbrace{\begin{pmatrix} \mu_0 \\ \mu_1 \\ \vdots \\ \mu_{d-1} \end{pmatrix}}_{\mu_S} dt + \underbrace{\begin{pmatrix} \sigma_{00} & 0 & \dots & 0 \\ \sigma_{10} & \sigma_{11} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ \sigma_{d-1,0} & \sigma_{d-1,1} & \dots & \sigma_{d-1,d-1} \end{pmatrix}}_{\Sigma_S} \underbrace{\begin{pmatrix} dB_t^0 \\ dB_t^1 \\ \vdots \\ dB_t^{d-1} \end{pmatrix}}_{dB_t^S}, \quad (1)$$

where  $(B_t^0, \dots, B_t^{d-1})$  are independent Brownian motions under  $P$ . The matrix of diffusion  $\Sigma_S$  is constant, positive definite, and invertible. Using the Itô's lemma, we can show that the vector of prices,  $S_t = (S_t^0, S_t^1, \dots, S_t^{d-1})^\top$ , satisfies the following relation:

$$d \ln S_t = \left( \mu_S - \frac{\text{diag}(\Sigma_S \Sigma_S^\top)}{2} \right) dt + \Sigma_S dB_t^S.$$

The asset  $S_t^0$  is the numeraire: the present value at time  $t$  of a cash-flow  $F$  paid at time  $T$  is equal to  $\mathbb{E}^Q \left( \frac{S_t^0}{S_T^0} F | \mathcal{F}_t \right)$ , where  $Q$  is a risk neutral measure. The numeraire is e.g. a short term rolling bond. In this case,  $\mu_0$  is the instantaneous interest rate (under the real measure  $P$ ). Investments of the insurance company are continuously rebalanced and proportions invested in each asset are summarized by the vector  $\theta_S = (\theta_0, \dots, \theta_{d-1})^\top$ . The total asset, denoted by  $A_t$ , is equal to  $\theta_S^\top S_t$  and its dynamics is defined by:

$$\frac{dA_t}{A_t} = \theta_S^\top \mu_S dt + \theta_S^\top \Sigma_S dB_t^S.$$

From the Itô's lemma, we infer that the total asset value is a lognormal random variable:

$$A_t = A_0 \exp \left( \left( \theta_S^\top \mu_S - \frac{1}{2} \theta_S^\top \Sigma_S \Sigma_S^\top \theta_S \right) t + \int_0^t \theta_S^\top \Sigma_S dB_s^S \right).$$

Notice that the financial market is incomplete. To underline this point, let us consider a change of measure

$$\left. \frac{dQ^{\mu^Q}}{dP} \right|_t = \exp \left( -\frac{1}{2} \int_0^t \chi^\top \chi ds - \int_0^t \chi^\top dB_s^S \right),$$

where  $\chi = \Sigma_S^{-1} (\mu_S - \mu^Q \mathbf{1}_{d-1})$  and  $\mu^Q \in \mathbb{R}^+$  is an arbitrary constant. Under  $Q^{\mu^Q}$ , the drift of all assets, including the numeraire, is equal to  $\mu^Q$  and discounted assets prices are martingales. This entails that  $Q^{\mu^Q}$  is a risk neutral measure, whatsoever the value chosen for  $\mu^Q$ .<sup>1</sup> In practice, actuaries set  $\mu^Q$  to the current risk free rate  $\mu_0$  but nothing prevents us (in our framework) from

<sup>1</sup> The incompleteness of the financial market results from the absence of pure discount bond. If a discount bond of maturity  $T$  is defined as  $B(t, T) = \mathbb{E}^Q \left( \frac{S_t^0}{S_T^0} | \mathcal{F}_t \right) = \exp \left( -\int_t^T \mu^Q ds \right)$ , then we can infer from the observation of its price the value of  $\mu^Q$ .

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