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Modeling trend processes in parametric mortality models[☆]

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ABSTRACT

Parametric mortality models like those of Lee and Carter (1992), Cairns et al. (2006), or Plat (2009) typically include one or more time dependent parameters. Often, a random walk with drift is used to project these parameters into the future. However, longer time series of historical mortality data often show patterns which a random walk with drift is highly unlikely to generate. In fact, historical mortality trends often appear to be trend stationary around piecewise linear trends with changing slopes over time (see, e.g., Sweeting (2011) or Li et al. (2011)). Periods of lower (but rather constant) mortality improvements are followed by periods of higher improvements and vice versa.

In this paper, we propose an alternative trend process which builds on the patterns observed in the historical data. Future trend changes occur randomly over time, and also the trend change magnitude is stochastic. Furthermore, we show how the parameters of this trend process, in particular the probability of observing a trend change in a certain year and the distribution for the trend change magnitude, can be estimated from historical data. We also outline how uncertainty in the parameter estimates can be accounted for. Finally, we compare the trend process to other trend processes which have been proposed in the literature.

1. Introduction

Longevity risk, i.e. the risk of insured or pensioners living longer than expected, has gained considerable attention over the last decades. The evolution of an increasingly active market for longevity risk transfers illustrates this. In order to measure longevity risk in annuity or pension portfolios, stochastic mortality models are required, and an enormous number of such models and model variants have been developed over the last decades. Most of them have a parametric structure which includes one or more time dependent parameters (period effects) to describe the evolution of mortality over time. In order to generate stochastic forecasts of future mortality, these parameters are projected into the future using stochastic processes. Obviously, it is crucial that these processes adequately project both the best estimate mortality evolution and the uncertainty of this evolution. Otherwise, risk management decisions will be based on deficient information, capital requirements will too high or too low, and longevity transactions will not be priced reasonably.

Fig. 1 shows, exemplarily, the two period effects κ_t^1 and κ_t^2 in the model of Cairns et al. (2006, CBD model) for English and Welsh males.[1] As we can see from the evolution of κ_t^1, mortality has been generally decreasing over the last 173 years. The parameter κ_t^2 describes the increase of mortality with age in year t, and we can infer from its overall increase over time that mortality improvements have been stronger for younger ages than for older ages in general. For projecting $\kappa_t = (\kappa_t^1, \kappa_t^2)$ into the future, a two-dimensional random walk with drift is used in most cases, i.e.

$$\kappa_t = \kappa_{t-1} + d + CZ, \quad (1)$$

where d is a time constant drift vector, Z is a vector of standard normal innovations, and C is an upper triangular matrix with $V = CC'$ being the covariance matrix of the innovations.

The (one- or multi-dimensional) random walk has been a very popular choice for projecting the period effects in other stochastic mortality models as well. One reason for that certainly is its simplicity. In its two-dimensional form, only five parameters need to be estimated from the historical data, i.e. the two elements of the drift vector d and the three entries in the matrix C which determines the volatility of the innovation vector. However, the

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[1] The CBD model is fitted to data from the Human Mortality Database (2015) for ages 50 to 89. For details on the model and the estimation of its parameters, we refer to Appendix A.

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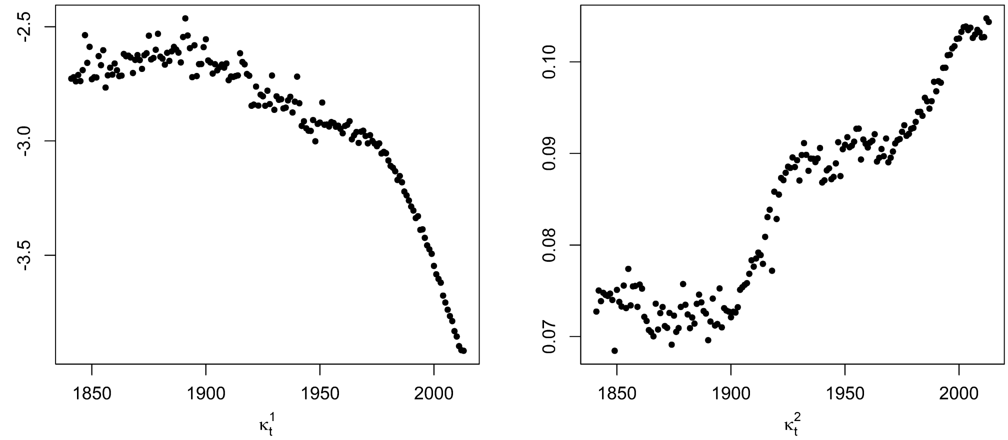

Fig. 1. Period effects in the CBD model for English and Welsh males.

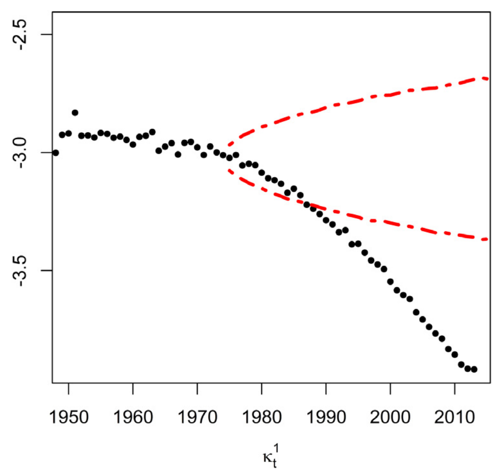

Fig. 2. Back test for period effect κ_t^1 in the CBD model for English and Welsh males; the period effect is projected by a random walk with drift and the dashed lines show the 99% confidence interval.

random walk's simplicity can also be problematic. Looking at data for the most recent decades only in Fig. 1, the assumption of a time constant drift appears reasonable. However, looking farther into the past, the trends in the period effects seem to have changed several times. This observation can be made for basically any population.

Thus, a constant drift seems to be a reasonable assumption only for a limited period of time. When projecting the period effects into the far future, the possibility of further trend changes should be taken into account — which the random walk does not. Potential trend changes in the future imply that the confidence bounds generated by a random walk with drift might be too narrow in the long run (see, e.g., Lee and Miller, 2001). Fig. 2 illustrates this by a back test in which a (one-dimensional) random walk is fitted to the κ_t^1 for English and Welsh males from 1956 to 1975 and then projected into the future.[2] In the long run, the realized κ_t^1 lie far outside even the 99% confidence interval. Börger et al. (2014) make an analogous observation for Dutch males.

Due to the random walk's structure, the width of a confidence interval it generates only depends on the volatility of the innovations. This volatility is fitted to annual random fluctuations in the historical data, and therefore, small (large) short term fluctuations automatically imply that long term trend uncertainty is also small (large). However, this is not reasonable in any case as the example of Liechtenstein and Switzerland shows. Due to Liechtenstein's population size, volatility is significantly larger than in Switzerland, and this also implies a larger parameter uncertainty in the projection. However, one would expect the long term trend uncertainty to be similar for both countries because of their very close political, economic, and social links. Thus, there is not necessarily a direct connection between short term volatility and long term trend uncertainty. Furthermore, Fig. 1 suggests that annual random fluctuations are instead trend stationary around piecewise linear trends.

For the aforementioned reasons, we believe that the general adequacy of the random walk with drift for projecting period effects in parametric mortality models is questionable, at least for long term projections. In fact, Fig. 1 indicates that a trend process should have the following properties:

1. Random fluctuations are stationary around some underlying trend.
2. The underlying trend evolves continuously and is piecewise linear.
3. The slope of the underlying trend can change at random points in time and in both directions.

In this paper, we derive such a trend process. We first consider a one-dimensional version before we then discuss its generalization to a multi-dimensional version as required, e.g., for the CBD model. We also show how the parameters of the trend process can be estimated from historical data and how parameter uncertainty can be accounted for. To this end, we apply the method proposed by Muggeo (2003) to fit a continuous and piecewise linear curve to historical data. From the thus detected historical trend changes we can estimate the probability of observing a trend change in a certain year in the future as well as a distribution of its magnitude. Furthermore, we can assess the uncertainty in these estimates. Finally, we compare our trend process to other trend processes which have been proposed in the literature. Even though we mostly focus on the CBD model in the examples and applications in this paper, it is important to note that our trend process can be applied within basically any parametric mortality model. The CBD model is just a convenient choice as it is possibly the most simple of all multi-dimensional mortality models. Moreover, it does not include any time constant parameters as, e.g., in the mixed time and age term $\beta_x \cdot \kappa_t$ in the Lee and Carter (1992) model where the assumption of constant β_x over longer time horizons is questionable.

2 The length of the estimation period is rather arbitrary, but 20 years seems to be a usual choice.

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