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Dynamic derivative-based investment strategy for mean–variance asset–liability management with stochastic volatility

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HIGHLIGHTS

- A mean–variance ALM problem with SV and a derivative is considered.
- The optimization problem is solved by adopting a BSDE approach.
- Explicit expressions of efficient strategies and efficient frontiers for three cases are obtained.
- Some numerical illustrations and interesting phenomena are provided.
- Derivatives are demonstrated to be a feasible tool to improve investment efficiency and reduce risk.

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ABSTRACT

This paper considers the derivative-based optimal investment strategies for an asset–liability management (ALM) problem under the mean–variance criterion in the presence of stochastic volatility. Specifically, an asset–liability manager is allowed to invest not only in a risk-free bond and a stock, but also in a derivative, whose price depends on the underlying price of the stock and its volatility. By solving a system of two backward stochastic differential equations, we derive the explicit expressions of the efficient strategies and the corresponding efficient frontiers in two cases, with and without the derivative asset. Moreover, we consider the special case of an optimal investment problem with no liability commitment, which is also not studied in the literature. We also provide some numerical examples to illustrate our results and find that the efficient frontier of the case with the derivative is always better than that of the case without the derivative. Moreover, under the same variance, the expectation of the case with the derivative can reach up to as twice as that of the case without the derivative in some situations.

1. Introduction

Asset–liability management (ALM) is one of the classic problems in the field of financial risk management. Typically, ALM involves the management of assets in such a way as to earn adequate return while maintaining a comfortable surplus of assets over existing and future liabilities. This problem is faced by a wide range of financial institutions, such as pension funds and insurance companies. Pre-commitment strategy of the mean–variance ALM problem has recently been studied widely. These studies consider an optimization problem of selecting an optimal portfolio that can yield sufficient return (by maximizing the expectation of the terminal surplus) in compensating the company's liability while minimizing risk measured by the variance of the terminal surplus, in which the underlying surplus is considered to be equal to the difference of liability from the asset. Leippold et al. (2004) considered a multi-period ALM problem under the mean–variance criterion, and derived explicit expressions for the optimal strategy and efficient frontier. By employing the stochastic linear–quadratic control theory, Chiu and Li (2006) investigated a continuous-time ALM problem under the mean–variance criterion and with the assumption that the risky assets' prices and the liability value were both governed by geometric Brownian motions. Xie et al. (2008) described the liability by a Brownian motion with a drift. Chen et al. (2008) and Chen and Yang (2011) extended the works of Chiu and Li (2006) and Leippold et al. (2004) to the case with Markovian

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regime switching market. Chiu and Wong (2012, 2013) applied the backward stochastic different equation (BSDE) method to study mean–variance ALM problems with cointegrated risky assets. Yao et al. (2013a, b) considered a continuous-time mean–variance ALM problem and a multi-period mean–variance ALM problem with uncertain time-horizon, respectively. Chiu and Wong (2014) investigated a mean–variance ALM problem with asset correlation risk, which was modeled by a multivariate Wishart process. The constant elasticity of variance model was studied by Zhang and Chen (2016), and the optimal portfolio strategy and efficient frontier were expressed by the solution of a BSDE. Pan and Xiao (2017a) incorporated stochastic interest rates and inflation risks into an optimal mean–variance ALM problem. Recently, some literature focuses on the time-consistent strategies of mean–variance ALM problems. For example, Wei et al. (2013) and Wei and Wang (2017) studied the time-consistent strategies of mean–variance ALM problems under the Markov regime-switching model and random coefficients setting, respectively. Zhang et al. (2017) investigated an ALM problem with state-dependent risk aversion under the mean–variance criterion. Besides the mean–variance criterion, the expected utility maximization is another important objective in ALM problems. Pan and Xiao (2017b, c) investigated an optimal ALM problem under the expected utility maximization framework with inflation risks and liquidity constraints, respectively. Chiu (2017) derived a new ALM solution for maximizing the expected utility subject to cointegrated assets and compound Poisson-type liabilities.

Although the ALM problem has been extensively investigated in various scenarios, two aspects are worthy of being further explored and compared with the aforementioned literature. On the one hand, few papers on the ALM problems include derivatives as investment opportunities. However, investment in derivatives is now attracting considerable interest thanks to their effectiveness in hedging financial risks. Financial institutions are using extensively various financial derivative instruments, such as swaps, options, forwards, credit derivatives and exchange-traded financial futures, in their trading and hedging strategies to align with the institutions' overall risk management strategy. According to a report from the Singapore Exchange (SGX) published on January 6, 2015, the value of securities trading fell by 25%, while the trading volume of derivatives rose to a record high in 2014. The quarterly report on bank trading and derivative activities of Office of the Comptroller of the Currency (OCC) showed that the credit exposure from derivatives increased in the first quarter of 2016, and the net current credit exposure (NCCE) from derivatives increased by \$65.1 billion or 16.5%, to \$460.1 billion. The annual report of Macquarie Group Limited for the financial year ended 31 March 2016 showed that the derivatives investment business in 2016 was about \$17.983 million, accounting for about 9.1% of the total assets. Therefore, it is highly relevant to incorporate the investment of derivatives into the ALM problems. Meanwhile, in the last decade, some scholars have focused on the investment of derivatives in portfolio selection. For example, Liu and Pan (2003) introduced derivatives to a portfolio selection problem, and pointed out that derivatives are essential for improving investors' welfare. Ilhan et al. (2005) investigated an optimal investment problem of an investor who maximized the expected exponential utility from the terminal wealth, combining a static position in derivatives with a traditional dynamic trading strategy in stocks. Hsuku (2007) studied a dynamic consumption and asset allocation problem with derivative securities under a recursive utility function. Jalal (2013) derived dynamic option-based investment strategies for a loss averse investor. Fu et al. (2014) considered a portfolio optimization in a continuous-time regime-switching market with derivatives. Escobar et al. (2015) considered an optimal investment strategy for an ambiguity-averse investor who had access to stock and derivative markets. Zeng et al. (2017) derived a derivative-based optimal investment strategy for an ambiguity averse pension plan member who faced risks from not only time-varying income and market return volatility but also uncertainty of economic condition over a long-time horizon.

On the other hand, few papers on the ALM problems with the mean–variance criterion considers the optimal pre-committed strategies (global optimal strategies) under the stochastic volatility model. The main reason is that there is a technical difficulty for the mean–variance problem under stochastic volatility model, and it is difficult to derive the closed-form solution, without which the verification theorem cannot be provided. However, stochastic volatility model can explain many stylized facts of financial markets, including the volatility smile and heavy-tailed nature of return distributions. Therefore, it is worthwhile to use some advanced mathematical tools and derive pertinent results for the mean–variance ALM problems with stochastic volatility. Applying the similar method used in Shen and Zeng (2015), we obtain the explicit expression of optimal investment strategy in an ALM problem with stochastic volatility and a derivative under the mean–variance criterion. Both Shen and Zeng (2015) and this paper focus on the mean–variance problem with stochastic volatility, and the main difference between this paper and Shen and Zeng (2015) is that Shen and Zeng (2015) considers an optimal reinsurance and investment problem without the derivative investment, while this paper studies an ALM problem and investigates the effect of the derivative investment. In fact, a system of two BSDEs is needed in this paper, while only a one-dimensional BSDE is used in Shen and Zeng (2015); in this paper, the second BSDE is a linear one, and its driver depends on the solution to the first BSDE and the liability process, both of which are unbounded. This makes the ALM problem in this paper more technically challenging. Furthermore, a comparison of Shen and Zeng (2015) and this paper shows that the variance risk can be fully hedged in the reinsurance–investment problem, but not in the ALM problem, although the financial markets in both papers are incomplete. This observation affirms that in the ALM problem, the inclusion of the derivative in the overall portfolio is more relevant and meaningful from the perspective of risk management.

To the best of our knowledge, there is no existing literature that addresses the ALM problem in the presence of derivatives as well as stochastic volatility. So this study is more meaningful from both theoretical and practical perspectives. We formulate a novel model to describe this problem based on existing literature about ALM. Specifically, in our model, the asset–liability manager is allowed to invest in a financial market consisting of not only a bond and a stock, but also a derivative. The stock price follows a stochastic volatility process, and the derivative is an option, the price of which depends on the stock price and the volatility of the stock. The liability process is described by a general stochastic process with hedgeable risks and unhedgeable ones. Under the mean–variance criterion, we express the efficient strategies and efficient frontiers in terms of the unique solution to a system of two BSDEs associated with the ALM problem.

The main contributions of this paper are as follows: (i) We pioneer to incorporate a derivative into the mean–variance asset allocation and ALM problems, which, to some extent, extends some existing papers on asset allocation and ALM problems, such as Chiu and Li (2006), Xie et al. (2008), Chiu and Wong (2014), Zhang and Chen (2016), and etc.; and the two cases, with and without derivative investment, are considered. (ii) A more general liability model is considered, in which the liability depends on not only the risks from financial market (price and stochastic volatility of the stock), but also other risk sources; this makes our model more practical. (iii) We find that derivatives can improve the performance of investment significantly; the efficient frontier in the case