Accepted Manuscript

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Please cite this article as: Sordo M.A., Bello A.J., Suárez-Llorens A., Stochastic orders and co-risk measures under positive dependence. *Insurance: Mathematics and Economics* (2017), https://doi.org/10.1016/j.insmatheco.2017.11.007

Stochastic orders and co-risk measures under positive dependence

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November 21, 2017

Abstract

Conditional risk measures (or co-risk measures) and risk contribution measures are increasingly used in actuarial portfolio analysis to evaluate the systemic risk, which is related to the risk that the failure or loss of a component spreads to another component or even to the whole portfolio: while co-risk measures are risk-adjusted versions of measures usually employed to assess isolate risks, risk contribution measures quantify how a stress situation for a component affects another one. In this paper, we provide sufficient conditions under which two random vectors could be compared in terms of CoVaR (conditional value-at-risk), CoES (conditional expected shortall) and different risk contribution measures. Conditions are given in terms of the increasing convex order, the dispersive order and the excess wealth order of the marginals under some assumptions of positive dependence.

1 Introduction

Consistency of risk measures with respect to stochastic orderings is an important topic in actuarial theory. Given two risks X and Y with respective distribution functions F and G, the literature on univariate risks contains many results of the form

$$X \prec_P Y \text{ implies } \rho\left[X\right] \leq \rho\left[Y\right], \text{ for all } \rho \in C, \tag{1}$$

where $\rho\left[X\right]$ and $\rho\left[Y\right]$ are two risk measures belonging to certain class C and $\prec_P$ denotes the ordering that is induced by some distributional comparison principle P. These results are of interest, for example, in the study of parametric families of risk distributions. Since these families are often ordered under different stochastic orders in terms of their parameters, results of the form (1) provide a way to know how changes in the parameters affect the risk of losses

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