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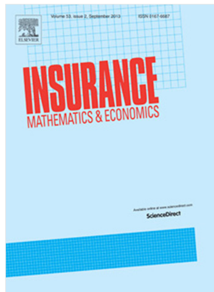

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Model Spaces for Risk Measures

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Abstract

We show how risk measures originally defined in a model free framework in terms of acceptance sets and reference assets imply a meaningful underlying probability structure. Hereafter we construct a maximal domain of definition of the risk measure respecting the underlying ambiguity profile. We particularly emphasise liquidity effects and discuss the correspondence between properties of the risk measure and the structure of this domain as well as subdifferentiability properties.

Keywords: Model free risk assessment, extension of risk measures, continuity properties of risk measures, subgradients
MSC (2010): 46B42, 91B30, 91G80

1 Introduction

There is an ongoing debate on the *right* model space for financial risk measures, i.e. about what an ideal domain of definition for risk measures would be. Typically—as risk occurs in face of randomness—the risks which are to be measured are identified with real-valued random variables on some measurable space $(\Omega, \mathcal{F})$. The question which causes debate, however, is which space of random variables one should use as model space.
Since *risk* is often understood as Knightian [21] uncertainty about the underlying probabilistic mechanism, many scholars argue that model spaces should be robust in the sense of not depending too heavily on some specific probabilistic model. We refer to this normative viewpoint as *paradigm of minimal model dependence.* The literature usually suggests one of the following model spaces:

(i) $\mathcal{L}^0$ or $L^0_{\mathbb{P}}$, the spaces of all random variables or $\mathbb{P}$-almost sure ($\mathbb{P}$-a.s.) equivalence classes of random variables for some probability measure $\mathbb{P}$ on $(\Omega, \mathcal{F})$, respectively, see [6, 7];

(ii) $\mathcal{L}^\infty$ or $L^\infty_{\mathbb{P}}$, the spaces of all bounded random variables or $\mathbb{P}$-a.s. equivalence classes of bounded random variables, respectively, see [6, 7, 15, 16, 23, 25] and the references therein;

(iii) $L^p_{\mathbb{P}}$, $p \in [1, \infty)$, the space of $\mathbb{P}$-a.s. equivalence classes of random variables with finite p-th moment, or more generally Orlicz hearts, see e.g. [3, 5, 17, 29].

The spaces in (i) and (ii) satisfy minimal model dependence in that $\mathcal{L}^0$ and $\mathcal{L}^\infty$ are completely model free, whereas $L^0_{\mathbb{P}}$ and $L^\infty_{\mathbb{P}}$ in fact only depend on the null sets of the probability measure $\mathbb{P}$. The problem with choosing $\mathcal{L}^0$ or $L^0_{\mathbb{P}}$, however, is that these spaces are in general too large to reasonably define aggregation based risk measures on them. The latter would require some kind of integral to be well-defined. Moreover, if $(\Omega, \mathcal{F})$ is not finite, $\mathcal{L}^0$ or $L^0_{\mathbb{P}}$

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