Optics and Lasers in Engineering 47 (2009) 747–753

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

# Assessment of measuring errors in DIC using deformation fields generated by plastic FEA

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ARTICLE INFO

*Article history:*
Received 4 February 2009
Received in revised form
4 March 2009
Accepted 4 March 2009
Available online 14 April 2009

*Keywords:*
Digital image correlation
Deformation measurement
Systematic errors
Finite element simulations

ABSTRACT

In this article, systematic errors that arise from different implementations of digital image correlation (DIC) techniques are analyzed. In particular, we investigate the influence of the adopted correlation function, the interpolation order, the shape function and the subset size on the derived displacements. These errors are estimated using numerically deformed images that were obtained by imposing finite element (FE) displacement fields on an undeformed image yielding plastic deformation of the specimen. This FE procedure simulates realistic experimental heterogeneous deformations at various load steps. It is shown that DIC is able to reproduce these displacements up to a satisfactory level if conscious choices in the above-mentioned implementations are made.



## 1. Introduction

Digital image correlation (DIC) offers unique opportunities for exploring full-field displacements and strain measurements in experimental mechanics. Indeed, DIC has shown to be an ideal tool to, e.g., identify the mechanical material behavior through inverse modeling [1,2] and to study the deformation characteristics of a wide range of materials [3–5].

In a first step of the experiment, a speckle pattern is usually attached to the specimen surface. The assumption that no cracks in this speckle structure appear during the deformation process is a fundamental prerequisite of the technique. Next, the DIC algorithm numerically compares two digital images of the specimen surface in the undeformed and deformed states, yielding displacement fields with sub-pixel accuracy. This sub-pixel registration process is one of the major challenges of DIC. Accordingly, the past two decades have resulted in the development of a number of DIC techniques in order to improve on this sub-pixel accuracy. Some DIC algorithms rely on the intensity interpolation [6–8], others on Newton–Raphson iteration [9,10], curve-fitting or interpolation of the correlation coefficients [11,12], optical flow method [13,14], complex spectrum [15], genetic and neural network methods [16,17] or the center of mass estimation of the correlation peak [3].

In this paper, we compute the sub-pixel displacements following the so-called subset-based method as described in Refs. [6,9,10]. In view of flexibility and in particular the possibility to freely implement additional features, we developed our own correlation software platform "*MatchID*". The validation of this code is performed in two ways. First, we test the ability of our software to reproduce a priori known rigid body displacements. A comparison to a commercial DIC system *Vic 2D* [18] is made. Next, we study two realistic complex experiments: an uni-axial tensile test on a perforated tensile specimen and a bi-axial tensile test on a perforated cruciform specimen. Reference images are numerically deformed by imposing finite element (FE) displacement fields. These are obtained by the commercial software package *Abaqus* [19], simulating the uni-axial and bi-axial tensile tests at various load steps resulting in substantial plastic deformation. Accordingly, we can validate our correlation predictions by comparing them to the well-known displacement fields at the FE nodes. In addition, we investigate the influence of the adopted correlation function, the interpolation order, the shape function and the subset size.

To our knowledge, this article is the first report of systematic errors in DIC in a realistic situation with large heterogeneous deformation regions. Previously, rigid body and quadratic displacement fields were the subject of investigation in Refs. [10,20–24], whereas Ref. [25] simulated uni-axial tension on a homogeneous specimen. On the other hand, in Ref. [26] a heterogeneous specimen is used, but only to study the accuracy of DIC related to the geometry of the speckle pattern.

We are well aware that additional errors may be introduced by the numerical deformation process. Care has been taken, however, in constructing the FE model such that the obtained displacement fields were independent from the element mesh, i.e. the element

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doi:10.1016/j.optlaseng.2009.03.007

size was sufficiently small that a further refinement of the mesh would result in the same values for displacements. It is the goal of this work, however, to check different implementations of the correlation algorithm, not to test the absolute accuracy of the subset-based method.

The outline of this article is as follows. In Section 2 we present the DIC formalism. Our results are included in Section 3. We conclude in Section 4.

## 2. Formalism

In a subset-based method, a matching between two speckle patterns is accomplished by considering a pixel and its neighborhood in the undeformed image $f$ and searching the same subset in the deformed image $g$, adopting a maximization routine for a similarity function. In general, the origin of an $(x, y)$ coordinate system is located at the upper-left corner of $f(x, y)$. Assign $(x_{sc}, y_{sc})$ as the coordinates of the center pixel of a $(2N+1)*(2N+1)$ subset in the image $f(x, y)$, with $N$ a positive integer number. Typical similarity functions are the cross-correlation coefficient $r_{CC}$ and the sum-of-squared-differences correlation coefficient $r_{SSD}$, defined as

$$r_{CC} = \frac{\sum_y \sum_x f(x,y) g[\mu(x,y,\mathbf{s}), v(x,y,\mathbf{s})]}{\sqrt{\left[\sum_y \sum_x f^2(x,y)\right]\left[\sum_y \sum_x g^2[\mu(x,y,\mathbf{s}), v(x,y,\mathbf{s})]\right]}}, \tag{1}$$

$$r_{SSD} = 1 - \frac{\sum_y \sum_x [f(x,y) - g[\mu(x,y,\mathbf{s}), v(x,y,\mathbf{s})]]^2}{\sum_y \sum_x f^2(x,y)}, \tag{2}$$

with $\sum_y = \sum_{y=y_{sc}-N}^{y_{sc}+N}$ and $\sum_x = \sum_{x=x_{sc}-N}^{x_{sc}+N}$.

The parameter vector

$$\mathbf{s} = \left[ u v \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} \frac{\partial v}{\partial y} \frac{\partial^2 u}{\partial x^2} \frac{\partial^2 u}{\partial y^2} \frac{\partial^2 v}{\partial x^2} \frac{\partial^2 v}{\partial y^2} \frac{\partial^2 u}{\partial x \partial y} \frac{\partial^2 v}{\partial x \partial y} \right] \tag{3}$$

relates coordinates in the reference image $f$ to the corresponding coordinates in the second image $g$ through

$$\begin{aligned} \begin{bmatrix} \mu(x,y,\mathbf{s}) \\ v(x,y,\mathbf{s}) \end{bmatrix} &= \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \\ &+ \begin{bmatrix} \frac{\partial^2 u}{\partial x \partial y} \\ \frac{\partial^2 v}{\partial x \partial y} \end{bmatrix} \Delta x \Delta y + \begin{bmatrix} \frac{\partial^2 u}{\partial x^2} & \frac{\partial^2 u}{\partial y^2} \\ \frac{\partial^2 v}{\partial x^2} & \frac{\partial^2 v}{\partial y^2} \end{bmatrix} \begin{bmatrix} (\Delta x)^2 \\ (\Delta y)^2 \end{bmatrix} \end{aligned} \tag{4}$$

with $\Delta x = x - x_{sc}$ and $\Delta y = y - y_{sc}$.

The way in which the subset can deform during the correlation process is defined by the number of parameters entering Eq. (4). Considering only $u$ and $v$, one reduces the problem to a rigid body one. In case also the first-order partial derivatives are taken into account, the general form of an affine transformation is retrieved. This latter accounts for rigid body motion (translation and rotation), shear and normal straining. Next, adding a term $\Delta x \Delta y$ yields a similar expression as the four node bilinear Lagrange interpolation function and allows the mapping onto an irregular quadrangle. Finally, second-order terms can be included in the shape function. In Ref. [27] it is stated that by accounting for these higher order gradients, the first order gradients and the displacement fields can be measured more accurately.

The purpose is now to find the parameter $\mathbf{s}_{\max}$ that maximizes the correlation coefficients of Eqs. (1), (2), or equivalently minimizes $C = 1 - r$. Developing $C$ into a second-order Taylor polynomial at a point $\mathbf{s}$ in the vicinity of the correlation peak, the maximum can be found at

$$\mathbf{s}_{\max} = \mathbf{s} + \Delta \mathbf{s}, \tag{5}$$

where the step $\Delta \mathbf{s}$ reads as

$$\Delta \mathbf{s} = -(\nabla^2 C)^{-1}(\nabla C). \tag{6}$$

Eqs. (5) and (6) are implemented in a Levenberg–Marquardt algorithm to iteratively find $\mathbf{s}_{\max}$.

In order to solve Eq. (6), gray value and gray value derivatives must be evaluated at noninteger pixel locations. Here, we adopt bilinear and bicubic polynomial interpolators. In Ref. [21] it is shown that in particular the transition from bilinear to bicubic order heavily improves on the accuracy. In addition, Ref. [21] introduces cubic and quintic B-spline interpolators, but a detailed discussion on this would fall beyond the scope of this work.

In Ref. [20] an additional approximation for the Hessian entering Eq. (6) is introduced. This approximation applies to the sum-of-squared-differences correlation coefficient $r_{SSD}$ of Eq. (2) and reads as

$$\frac{\partial^2 C}{\partial s_i \partial s_j} \approx \frac{2}{\sum_y \sum_x f^2} \sum_y \sum_x \frac{\partial g}{\partial s_i} \frac{\partial g}{\partial s_j}. \tag{7}$$

Further on, the results obtained via Eq. (7) will be labeled as approximate-sum-of-squared-differences ($r_{ASSD}$).

## 3. Results

The goal of this paper is to study systematic errors in DIC introduced by the adopted correlation function, the interpolation order, the shape function, subset size and so on, by using numerically deformed images simulating realistic plastic deformation with a large degree of heterogeneity. Before embarking on the study of those effects, however, we first validate our code by checking its ability to reproduce a priori known rigid body displacements.

### 3.1. Rigid body translation

Numerical simulations of rigid body translation are performed according to Refs. [21,22]. In particular, to safeguard the shifted images against phase or amplitude corruption, we apply a Fourier filter $\exp(-i0.05\pi n)$ on the initial undeformed manually generated speckle pattern displayed in the left panel of Fig. 3. This filter corresponds to a shift of 0.05 pixels between successive images, with $n$ referring to the number of the image. The simulated image, with a well-known rigid body displacement of $n * 0.05$ pixels, can then be compared to the undeformed image via DIC software.

Fig. 1 displays the difference between the imposed $u_{imp}$ and the mean DIC $u_{mean}$ horizontal components and their corresponding standard deviations for sub-pixel displacements ranging from 0 to 2 pixels. The mean values $u_{mean}$ are calculated using 8000 ($200 \times 400$) points per image. A subset size of $19 \times 19$ is adopted as displayed in the left panel of Fig. 3. The results are obtained via bicubic interpolation, the cross-correlation coefficient and affine transformation shape functions. Identical settings were adopted for the commercial *Vic2D* software, except for the correlation function which corresponds to the sum-of-squared-differences one. We observe that both results are in good agreement. Indeed, the prescribed sub-pixel displacements are reproduced with at least 0.04 pixel accuracy. For an elaborated discussion on the origin of the sinusoidal shape, the impact of the interpolation function on the systematic errors and the improvement of the accuracy, we refer to Ref. [21].