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Energy-aware lot sizing problem: Complexity analysis and exact algorithms

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ABSTRACT

The single-item lot sizing problem under a periodic energy limitation is considered in this paper. Identical and parallel capacitated machines constitute the production system, each one consuming a certain amount of energy when being switched on, when reserved, and when producing. We consider a cost for starting-up the machines, a reservation cost to keep the machines ready for production, in addition to classical lot sizing costs, such as, joint setup cost, unit production cost and unit holding cost, all being time-dependent. Besides the classical lot sizing decisions of how much and in which periods to produce, we have to decide the number of machines to switch on and to switch off in each period. We show that this problem is NP-hard even under restricted conditions. In contrast, assuming stationary energy parameters, we propose two polynomial time dynamic programming algorithms to solve the problem to optimality. The first algorithm is proposed for the case with null setup cost, null reservation cost and null reservation energy consumption, and runs in $O(M^5T^4)$ time, with M being the number of machines and T the number of periods. We show that we can extend this algorithm to solve the generalized version of the problem in time complexity $O(M^6T^6)$.

1. Introduction

Energy-efficiency in production planning becomes more and more appealing for researchers and practitioners. According to Biel and Glock (2016), "*In 2010, the industrial sector was responsible for 39.4% of the overall energy consumption and this latter largely originates from manufacturing industries.*" The same authors mention that the aim of energy-efficient production planning models is not only to take into account the classical metrics such as the minimization of overall cost or completion time, but also to consider energy-aware factors such as energy related constraints, energy cost or energy consumption minimization, etc. There is also a change in the consumers' behavior, with a higher sensitivity to the environmental impacts of the industrial activities (pollution, energy consumption, etc.). For couple of years, numerous companies have thus begun to rethink and optimize their production processes in order to produce at lower cost, but also more ecologically and with a lower energy consumption.

We consider in this paper a lot-sizing problem taking into account the energy consumption as a hard constraint, in a capacitated machines environment. In addition to a classical production capacity, dependent on the number of machines running in a given period (which is a decision variable in our problem), we consider a limit on the amount of energy that can be consumed in each period by the production system. The different activities responsible for consuming energy that we consider in this paper, include the start-up of the machines, the production of goods, and keeping the machines ready, either they do produce or are idle. The aim is to decide when and how much to produce, when and how many machines to turn on or to turn off, in order to minimize the total cost incurred over a time horizon of T periods, while respecting the amount of energy available in each period and satisfying the demand. We consider the deterministic version of the problem, that is, we assume that all the parameters and the demand to satisfy are known for each period of the time horizon. The cost of a planning includes a start-up cost to switch on the machines, a reservation cost to keep the machines ready for production, a joint setup cost, a unit production cost, and a unit holding cost to carry units in stock. We believe that the problem studied is in accordance with the context of energy aware production and environmental sustainability. We call this energy aware lot sizing problem *energy*-LSP in the rest of the paper.

Contributions of the paper. Problem *energy*-LSP in a parallel production system was first introduced by Rapine et al. (2018). They propose a very efficient $O(T \log T)$ algorithm for a restricted version assuming that start-up costs are stationary and that only one activity (start-up or production) consumes energy. In this article we extend the model in different directions to render it more realistic: First, we consider time-varying cost parameters, including start-up costs but also non

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null joint production setup costs, to be paid in each period where production occurs, and reservation costs, to be paid for each machine kept ready to produce. Second, we consider that all activities may have a non negligible energy consumption. It means that, in each period, one has to arbitrate how the available amount of energy is to be shared between the start-up of machines, which increases the production capacity of the system, and the effective production of units. In addition, we also consider a reservation energy consumption, that represents the energy consumed by a machine that is kept on, whenever producing or not. Under this quite general framework, we establish that the problem is NP-hard if some energy parameters are time-varying, even on a single resource with non-null setup or reservation costs. In contrast, we show that the problem is polynomially solvable if all the energy parameters, that is, periodic amount of available energy, start-up, reservation and unit consumptions, are stationary. Our approach is based on dynamic programming and provides an $O(M^6T^6)$ exact algorithm for the most general case studied in this paper. To the best of our knowledge, there is no other theoretical studies on this integrated lot sizing and energy issues, except (Rapine et al., 2018).

Organization of the paper. The paper is organized as follows. In Section 2, we present relevant studies in both lot sizing and energy-efficient production planning problems. The problem description is given in Section 3 via a mixed integer programming formulation for the general case. In Section 4, we establish that problem *energy*-LSP is NP-hard, even with null production and null holding costs. Then, several other complexity results are given for different restricted cases. In Section 5, we first give a polynomial time algorithm for the restricted version of the problem, with null setup cost, null reservation cost and null reservation energy consumption. Under stationary energy parameters, this algorithm solves the problem exactly in time complexity of $O(M^5T^4)$. In the same section, this algorithm is extended to solve *energy*-LSP with non-null reservation cost, joint setup cost and reservation energy consumption parameters, which results in a time complexity of $O(M^6T^6)$. We finally conclude in Section 6 with several perspectives.

2. Literature review

The problem studied in this article can be positioned at the intersection of the single-item dynamic lot sizing problem (LSP) and energy-efficient manufacturing. In this section, relevant studies published in both domains are presented, together with the very few studies at their intersection.

The single-item LSP aims to determine how much and in which periods to produce in order to satisfy a deterministic and discrete demand over a given time horizon, while minimizing the total production and storage costs. The reader can refer to Wagner and Whitin (1958) for a seminal paper, and to Florian et al. (1980) and to Bitran and Yanasse (1982) for the first complexity analysis on the capacitated LSP. For more details on the different extensions of this extensively studied production planning problem, we refer to Brahimi et al. (2006), Pochet and Wolsey (2006) and to Brahimi et al. (2017). In the literature, most of the existing problems in production planning focus on the minimization of production and holding costs. However, in order to respect the new environmental standards and energy consumption issues, more and more theoretical and practical applications integrate them within the optimization of the production planning (see (Gahm et al., 2016) and (Biel and Glock, 2016)).

The aim of this paper is to integrate the energy constraints into the lot sizing problem. In the related literature, some studies integrate explicitly the energy cost in the optimization problem (see Özdamar and Birbil (1999), Uzel (2004), Tang et al. (2012) and Ding et al. (2016)), but they assume that the available energy is unlimited. We propose in this paper an approach considering a given limit on the energy available in each period, which hence also limits the quantity that can be produced. Notice that a limit on the amount of available energy is also considered by Artigues et al. (2013), Nattaf et al. (2015), (Nattaf et al., 2016), Ngueveu et al. (2016) and Modos et al. (2017) for scheduling problems, by Schultz et al. (2015) for a short term production control problem and by Masmoudi et al. (2017) for a single-item capacitated LSP. In (Masmoudi et al., 2017), the authors consider a flow-shop system with a maximum allowable energy level, as well as an electricity price in their objective function. The flow-shop system considered in (Masmoudi et al., 2017) makes the problem quite different than ours with deadlines to respect. Moreover, the authors propose heuristics to solve their problem, while we theoretically study our models, analyzing their complexity and proposing polynomial time exact algorithms. A very recent study from Giglio et al. (2017) integrates energy consumption issues into the lot sizing and scheduling decisions in a multi-item, multi-machine job-shop environment, with additional backlogging and remanufacturing assumptions. The capacitated machines consume a certain amount of energy when being idle and when producing units, where a normal production mode and an accelerate production mode are distinguished. All the latter energy issues are modeled via costs of energy consumption into the objective function. Giglio et al. (2017) propose a relax-and-fix heuristic to cope with this integrated problem.

There are also several papers dealing with industrial case studies on energy issues in production planning problems (see (Artigues et al., 2013), (Santos and Almada-Lobo, 2012), (Waldemarsson et al., 2013) and (Zhao et al., 2016)). Those case studies provide us with a real motivation to integrate energy issues into the production planning problems. For instance, in Ozdamar and Birbil (Özdamar and Birbil, 1999), the authors provide some examples of energy-intensive industries, such as steel, tile and glassware industries, where, the energy expenditure represents the major component of the unit production cost. Tang et al. (2012) study a production planning problem for a hot rolling production process, where they try to minimize the energy consumption. Waldermarsson et al. (Waldemarsson et al., 2013) consider a pulp company, and they study an integrated supply chain planning taking energy issues into consideration. Zhao et al. (2016) study the scheduling of parallel furnaces and energy distribution for real world ethylene plant. All the above cited case studies show the importance of taking into account energy cost and limitation within operational planning in manufacturing. The reader can be referred to Gahm et al. (2016) for a recent review on energy-efficient scheduling issues in manufacturing, to Biel and Glock (2016) for a literature survey on the energy-efficient production planning, to May et al. (2017) for a detailed literature review on the energy management in manufacturing and finally, to Schulze et al. (2016) for energy management issues in industry.

The majority of the papers published in the domain of energy-efficient production planning consists in energy-efficient machine scheduling problems (see Biel and Glock (2016)). To the best of our knowledge, there are only a few studies in the literature coupling energy issues with discrete lot sizing problem: Masmoudi et al. (2017), Giglio et al. (2017), that we have already discussed, and Rapine et al. (2018). As mentioned earlier, the problem studied in this paper consists in an extension of the energy lot sizing problem presented in Rapine et al. (2018). The main differences between the two models are, first, the fact that in this paper, several activities, such as the production of units, the start-up of machines, and the reservation of machines consume a non negligible amount of energy. Second, we consider a more general cost structure, including reservation cost, joint setup cost, and start-up costs, all allowed to be time-dependent in our model. Our resolution approach, based on dynamic programming, is also totally different from the algorithm presented in Rapine et al. (2018).

3. Problem formulation

The system we study is constituted of M parallel, identical and capacitated machines that can be started at any period, respecting the energy restriction. Each machine has a constant capacity of U. We say