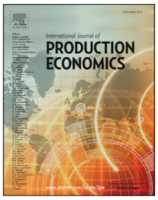
International Journal of Production Economics 200 (2018) 302–310

Contents lists available at ScienceDirect

International Journal of Production Economics

journal homepage: www.elsevier.com/locate/ijpe



Optimizing make-to-stock policies through a robust lot-sizing model

Agostinho Agra[a,*], Michael Poss[b], Micael Santos[c]

[a] CIDMA, Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal
[b] UMR CNRS 5506 LIRMM, Université de Montpellier, 161 rue Ada, 34392 Montpellier Cedex 5, France
[c] Department of Mathematics, University of Aveiro, 3810-193 Aveiro, Portugal

ARTICLE INFO

Keywords:
Lot-sizing
Make-To-Stock
Robust optimization
Mixed-integer linear programming

ABSTRACT

In this paper we consider a practical lot-sizing problem faced by an industrial company. The company plans the production for a set of products following a Make-To-Order policy. When the productive capacity is not fully used, the remaining capacity is devoted to the production of those products whose orders are typically quite below the established minimum production level. For these products the company follows a Make-To-Stock (MTS) policy since part of the production is to fulfill future estimated orders. This yields a particular lot-sizing problem aiming to decide which products should be produced and the corresponding batch sizes. These lot-sizing problems typically face uncertain demands, which we address here through the lens of robust optimization. First we provide a mixed integer formulation assuming the future demands are deterministic and we tighten the model with valid inequalities. Then, in order to account for uncertainty of the demands, we propose a robust approach where demands are assumed to belong to given intervals and the number of deviations to the nominal estimated value is limited. As the number of products can be large and some instances may not be solved to optimality, we propose two heuristics. Computational tests are conducted on a set of instances generated from real data provided by our industrial partner. The heuristics proposed are fast and provide good quality solutions for the tested instances. Moreover, since they are based on the mathematical model and use simple strategies to reduce the instances size, these heuristics could be extended to solve other multi-item lot-sizing problems where demands are uncertain.

1. Introduction

In this paper we consider a practical problem occurring in an aluminium extrusion industrial company. The company produces two main families of products: a family of products representing the main production activity of the company where a Make-To-Order (MTO) policy is followed (MTO family), and a family of products whose orders are typically quite below the established minimum production level. For this family, the company follows a Make-To-Stock (MTS) policy (MTS family). The production planning procedure for the MTO family is well established. However for the MTS family, as the orders are below the minimum production level, the company must find a solution between the two extreme cases: wait for new orders of the same product until the minimum production level is attained, or produce at least at the minimum production level of that item to satisfy the pending orders and store the leftovers in inventory. Both alternatives have their pros and cons. The first alternative has the advantage of avoiding stocks. On the other hand, the backlogging of demand orders may lead to intangible losses. Conversely, the second alternative has the advantage of a ready satisfaction of customer needs but generates high holding costs.

Currently, the company gives priority to the MTO family by planing its production first, and when extra production capacity is available, then it solves a lot-sizing problem to decide which products from the MTS family should be produced and defining the corresponding lot-sizes. This particular lot-sizing problem takes into account not only the pending orders of each product but also future ones, as the excess quantity produced will remain in stock until new orders are received. Therefore, it is necessary to estimate those future client orders. The uncertainty related to forecasting such future demands represents a risk for the planners since the inventory costs will depend greatly on such unknown demands. For industries where holding costs are high (as in the case of our industrial partner) it is desirable to derive robust solutions that take into account possible future deviations from the estimated demand values.

Here we address this lot-sizing problem defined for the MTS family of products, using the available production capacity. We consider both the deterministic and the robust cases where demands are assumed to belong to an uncertainty set and we look for the production plan that optimizes the worst-case scenario. For the production of the MTS family, we

---

* Corresponding author.
E-mail addresses: aagra@ua.pt (A. Agra), michael.poss@lirmm.fr (M. Poss), micael92@ua.pt (M. Santos).

https://doi.org/10.1016/j.ijpe.2018.04.002
Received 21 April 2017; Received in revised form 30 March 2018; Accepted 3 April 2018

produce at most one batch of each product, hence, we allow at most one set-up. Therefore this particular lot-sizing problem is denoted by LS1S(Lot-Sizing with 1 Set-up). The robust problem is denoted by RLS1S.

Multi-product lot-sizing problems have been receiving a great attention, for recent publications, see e.g. (Cunha et al., 2017; Macedo et al., 2016; Sifaleras and Konstantaras, 2017). Frequently, due to the variety of products and their demand patterns, the companies follow different production polices for the different products. In some cases, different policies can even be considered for the same product (see (Zhang et al., 2013)) in order to satisfy the different demand streams. The decision between the MTO and the MTS policies was investigated by Zaerpour et al. (2008) and Altendorfer and Minner (2014). For an overview on comparison of such approaches see (Olhager and Prajogo, 2012). However, both MTO and MTS producing processes may share common resources forcing the production planners to coordinate the MTO and MTS policies (Rafiei and Rabbani, 2012). Examples of problems combining MTO–MTS policies can be found in different industries, such as food production systems (Soman et al., 2004) and steel plants (Zhang et al., 2015).

Several approaches have been proposed, mostly from last decade, regarding the integration of MTS and MTO policies. Beemsterboer et al. (2016) study the benefits of not prioritizing policies within a hybrid planning MTO–MTS approach. In (Beemsterboer et al., 2017a), the authors analyse the benefits of considering flexible lot sizing policies in a hybrid MTO–MTS approach for a two-product system. In (Beemsterboer et al., 2017b), the authors propose four methods of integrating make-to-stock items in the control of a job shop, which they evaluate using discrete event simulation. Kaminsky and Kaya (2009) propose heuristics for a multi-item problem where the manufacturer and the supplier have to decide which items to produce to stock and which to produce to order. Kalantari et al. (2011) present a decision support system for order acceptance/rejection in a hybrid MTO–MTS production environment. Perona et al. (2009) develop a decision-making approach to support inventory management decisions in a MTO–MTS environment for small and medium sized enterprises. Renna (2016) considers a multistage manufacturing serial system, where a production control strategy is performed to release MTO and MTS orders. Rafiei et al. (2013) propose a hierarchical production planning approach for a hybrid MTO–MTS system that includes both mid-term and short-term production planning levels. Rafiei et al. (2014) propose a genetic algorithm for a multi-site production planning of a hybrid MTO–MTS manufacturing system.

The MTS planning carries the risk that the forecasted orders may not materialize. Such risk has been identified before, see (Tang and Musa, 2011). When it is possible, delaying product differentiation can be an interesting intermediate solution (Gupta and Benjaafar, 2004), but that is not possible in most practical cases as the one faced by our industrial partner. For those cases, handling with uncertainty is of main relevance on MTS environments. To the best of our knowledge only Khakdaman et al. (2015) applied a robust multi-objective approach based on a set of scenarios to a hybrid MTO–MTS problem where uncertainty is considered in suppliers, processes and customers.

The problem considered in this paper occurs as a subproblem of a hybrid MTO–MTS manufacture system where a hierarchic approach is followed and priority is given to MTO. The problem focuses on solving the MTS planning considering the remaining manufacturing capacity. From its nature, the MTS subproblem considers medium/long-term horizons where demand uncertainty plays a crucial role when defining lot-sizings.

A large number of publications has been devoted to the study of robust lot-sizing problems with demand uncertainty. One of the first papers on the topic is (Bertsimas and Thiele, 2006), which proposes a simple conservative approximation of the robust constraints and studies the structure of the optimal policies. In parallel to that work, another paper introduced affine decision rules (Ben-Tal et al., 2004), having the advantage of better approximating the robust constraints. The theoretical strength of affine decision rules has been studied in subsequent papers, among which (Iancu et al., 2013). More recent works have sought to solve the robust problem exactly, by using decomposition algorithms and dynamically adding constraints to the problem, see (Agra et al., 2016; Bienstock and Özbay, 2008; Gorissen and den Hertog, 2013). Robust lot-sizing problems and their variants are also addressed in more general papers dealing with multi-stage robust optimization, see (Delage and Iancu, 2015) for a survey on these problems. More generally, we refer to (Peidro et al., 2009) for a survey on papers dealing with uncertainty on supply chains.

Although motivated by a practical problem, we aim to incorporate the recent robust optimization techniques into this particular lot-sizing problem in order to close the gap between the robust techniques for classical lot-sizing problems and the robust techniques for MTS problems within hybrid MTO–MTS manufacture systems.

The contributions of this paper are more specifically detailed below. We introduce a mathematical model for the deterministic case where future demands are assumed to be known. Our model is different from the classical ones (see for instance (Pochet and Wolsey, 2006)) mainly because we suppose that each product has at most one set-up. A Proof that this particular problem is NP-hard is given. The model is tightened with valid inequalities.

We develop a robust mixed integer model where demands are considered uncertain and belong to intervals. The uncertainty set is further constrained by budget constraints that limit the number of possible periods where a demand can deviate from its nominal value preventing the solutions to be too conservative, obtaining the well-known budgeted uncertainty set introduced in (Bertsimas and Sim, 2004). We approximate the resulting robust constraints using the conservative approach of (Bertsimas and Thiele, 2006), rather than the computationally demanding affine decision rules from (Ben-Tal et al., 2004) or exact approaches used in (Agra et al., 2016; Bienstock and Özbay, 2008).

Since the problem is NP-hard, and we aim to develop approaches that can be used both with commercial and non-commercial (slower but free) solvers, we propose two heuristics. The first heuristic, called *Elite Heuristic*, is based on a pre-selection of a set of candidate products. The problem is solved for that restricted set of products using a mixed integer linear programming solver based on the strengthened formulation. The heuristic incorporates the practical rules used by the company to choose the products to produce. The second heuristic, denoted as the *Tournament Heuristic*, runs in several iterations. At each iteration, the set of candidate products is partitioned into smaller subsets and the problem is solved optimally for each subset. Only the selected products of each subset are considered in the next iteration. The process is repeated until a final subset of products is solved or a number of iterations is attained.

To test the deterministic and robust formulations and the matheuristics we use the non-commercial solver Cbc from Coin-OR (2016), which is referred to as one of the fastest solvers among the non-commercial ones (Meindl and Templ, 2012). The test set was built from the real data provided by our industrial partner.

As the proposed heuristics use simple strategies to reduce the number of items and, consequently, the size of the instances, such heuristics can be easily adapted to other multi-item lot-sizing problems. It suffices to adapt the mathematical model to the particularities of the other problems. We also show, that in order to derive solutions that take into account future demands variations, robust strategies could be embedded into the mathematical model, and therefore into the heuristics, but of course such strategies would need further computational testing in other cases and contexts.

The outline of the paper is as follows. In Section 2 we introduce a mixed-integer formulation to model the practical LS1S problem assuming the demands are deterministic. The formulation is enhanced and a Proof of NP-hardness is given. Then, in Section 3, we derive the robust model for the case where demands belong to an uncertainty set. In Section 4 we