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Analysis of white-light interferograms by using Stockwell transform

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ABSTRACT

This paper proposes the use of Stockwell transform for the analysis of white-light interferograms. The performance of Stockwell transform is assessed from the statistical parameters obtained by analyzing the simulated and experimental interferograms. Furthermore, the sensitivity of Stockwell transform to sampling, intensity and the phase noises is investigated. Results show that sampling and intensity noises significantly affect the performance of Stockwell transform.

1. Introduction

White-light interferometer (WLI) is an optical method to measure the surface profiles and roughness [1–5]. Interferogram analysis is one of the most important aspects in interferometery. Signal processing algorithms are generally developed by using ideal white-light interferograms (correlograms). However, if they are applied to experimental correlograms obtained from the setup, the results cannot be found accurately. Besides, the phase and the shape of the correlogram mainly change due to the dispersion and the noise in the system [9]. So far, the commonly used algorithms for the analysis of interferograms are sliding average, contrast, fast Fourier transform (FFT) and continuous wavelet transform (CWT) methods [5–9].

In this study, a new approach based on Stockwell transform (ST) [10] is proposed to analyze the correlograms. Although, it has been applied to several problems in Electrical Engineering and Physics, this is the first time the method is used to evaluate the interferograms. The aim of this work is to investigate the results of application of the ST to such optical problems. The sensitivity of the ST algorithm to noise and systematic errors is determined by the statistical parameters obtained from the evaluated results.

2. The analysis of the synthetic correlograms by using the ST algorithm

White-light interferogram (correlogram) as a function of optical path difference (OPD) z is given by Recknagel et al. [8] as

$$I(z) = I_0 + I_1\,\cos(kz + \varphi)\,e^{-4(z-z_0/l_c)^2} \tag{1}$$

where I_0, $k = 2\pi/\lambda$, $\bar{\lambda}$, l_c, I_1, φ z_0 are the mean intensity, wave number, the mean wavelength, the coherence length, the modulation amplitude, phase shift of the carrier frequency and the envelope peak position, respectively.

ST can be derived by using the CWT. The CWT of the correlogram in (1) is given by [10,11]

$$W(a,b) = \int_{-\infty}^{+\infty} I(z) M*(z-b,a)\,dz \tag{2}$$

where $M(z{-}b,a)$ is a mother wavelet in which b and a are the translation and the scale parameters, respectively.

The ST can be defined as a CWT with a specific mother wavelet multiplied by a phase factor [10]. It can be expressed as

$$S(f,b) = e^{i2\pi fb} W(a,b) \tag{3}$$

where f is the frequency which equals the inverse of α. The mother wavelet given for the ST can be written as

$$c(z,f) = \frac{|f|}{\sqrt{2\pi}}\,e^{-i2\pi fz}\,e^{-z^2f^2/2} \tag{4}$$

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The mother wavelet depending on the parameter b can be rewritten as

$$c(b-z,f)=\frac{|f|}{\sqrt{2\pi}}\,e^{-i2\pi f(b-z)}\,e^{-(b-z)^2f^2/2} \tag{5}$$

As a result of this, the ST of the correlogram $I(z)$ can be given by

$$S(f,b)=\int_{-\infty}^{+\infty} I(z)\frac{|f|}{\sqrt{2\pi}}\,e^{-(b-z)^2f^2/2}\,e^{-i2\pi fz}\,dz \tag{6}$$

The proposed method uses the amplitude of the S transform ($|S(f,b)|$) to process the correlogram. The highest values of the amplitudes for each frequency are determined. Based on these values, the position for the largest peak value corresponds to the height value of the correlogram (z_0).

Next, an ideal correlogram will be simulated, and the proposed method will be tested. During the simulations, the OPD z is varied from 0 to 20 μm and the mean wavelength and the coherence length are considered to be $\bar{\lambda}=754$ nm and $l_c=3$ μm, respectively. Also, the system scan speed with a video camera at 50 Hz frame rate is 4 μm/s, and z_0 is 10 μm. The ST of a simulated ideal correlogram is depicted in Fig. 1b. An ideal correlogram (see Fig. 1a) and its height position are given in Fig. 1c.

Moreover, the sensitivity of the ST algorithm to noise and systematic errors is tested with some simulated noisy correlograms. These noisy correlograms are obtained by using (7)

$$I(z)=I_0+I_1\cos(k(z-z_s)+\varphi_s)\,e^{-4(z-z_0-z_s/l_c)^2}+n_I \tag{7}$$

where z_s represents the sampling noise created by the translation stage in the experimental setup. n_I shows the intensity noise created by the system. φ_s gives the phase noise due to dispersion and other effects.

Firstly, the intensity noise is considered, and several correlograms including white Gaussian noise (WGN) are simulated. This noise is obtained within 10–30 dBW by using Matlab signal processing toolbox. Some of these noisy correlograms which have the noise powers of 10, 20 and 30 dBW are analyzed by using the ST. To show the effect of the intensity noise on the white-light interferogram, the correlogram having the noise power of 30 dBW is given in Fig. 2a. Its ST is depicted in Fig. 2b. This noisy correlogram and its calculated z_0 value are shown in Fig. 2c. Besides, the results for the correlograms with noise powers of 10, 20 and 30 dBW are given in Table 1. The effect of the intensity noise (30 dBW) on both the correlogram and its height value can be seen in Fig. 2. It is observed that the correlogram is destroyed by the random intensity noise. Thus, the z_0 value of the correlogram cannot be calculated accurately by the algorithm for the noise power of 30 dBW. The ST gives its height value with the RMS error of 8×10^{-2}. However, it is seen in Table 1 that the z_0 values are calculated correctly in case of 10 and 20 dBW noise powers.

Secondly, to study the effect of sampling noise on the performance of ST algorithm, some correlograms having z_s are simulated while varying the sampling noise from 50 to 200 nm. The correlogram with the sampling noise of 200 nm is shown in

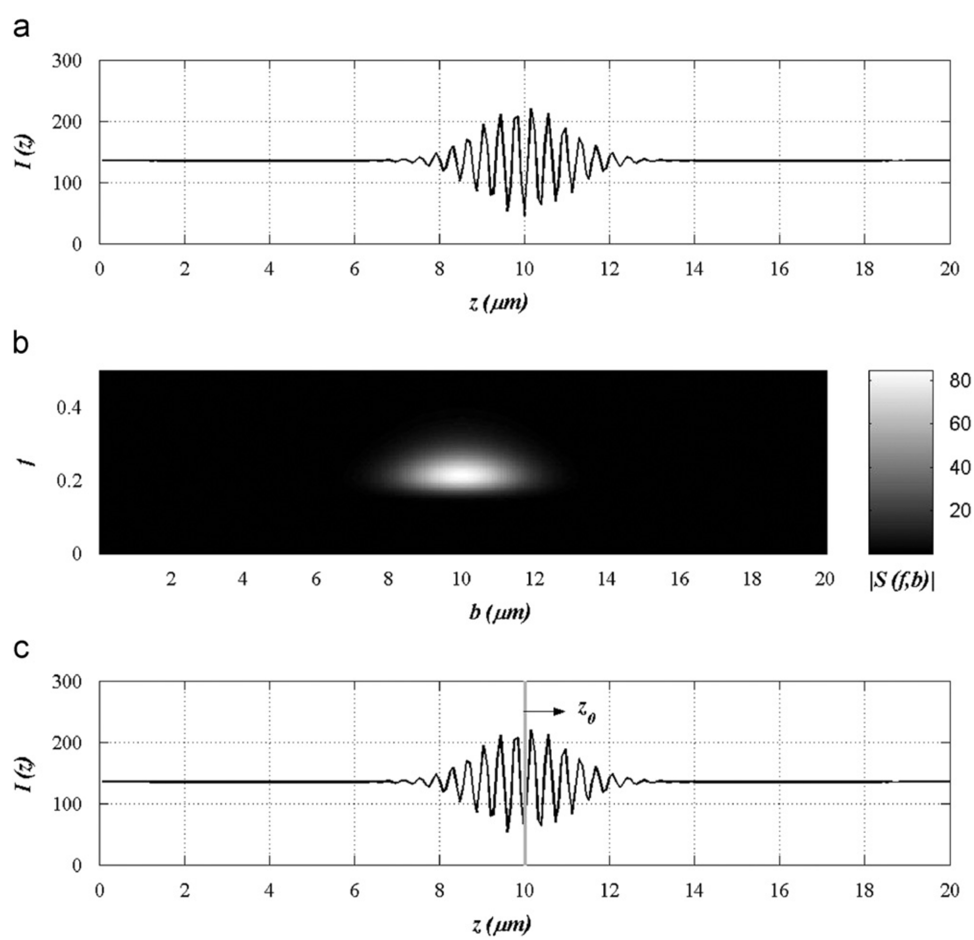

Fig. 1. (a) A simulated ideal correlogram, (b) the ST of a simulated ideal correlogram and (c) an ideal correlogram in (a) and its height value.