ARTICLE IN PRESS

International Review of Economics and Finance xxx (2017) 1–17

Contents lists available at ScienceDirect

International Review of Economics and Finance

journal homepage: www.elsevier.com/locate/iref

Pricing and hedging barrier options under a Markov-modulated double exponential jump diffusion-CIR model

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ARTICLE INFO

Keywords:
Hidden Markov chain
Double exponential jump diffusion process
Barrier options

ABSTRACT

A semi-closed-form valuation model is presented for barrier options whose underlying asset follows a mean-reverting and regime-switching double exponential jump diffusion process, and the interest rate is modulated by a mean-reverting square root model. The proposed model captures the impact of regime-switching uncertainty on barrier option prices and their hedge parameters in long and short business cycles. The model provides richer economic insight and is more appropriate for valuing barrier options in commodity markets as well as in equity and foreign-exchange markets, when an economy faces regime-switching uncertainty.

1. Introduction

A European barrier option's payoff depends on whether the underlying asset price has crossed a predetermined barrier before the exercise time. For example, an up-and-in call (UIC) option pays the usual European call payoff if the underlying asset price touches the barrier level any time before the option's expiry. Because barrier options are cheaper than regular European options, they are attractive to investors and hedgers who are averse to paying high premiums. Additionally, sellers of barrier options may be able to limit their downside or upside risk. Other related barrier options have become widely employed in global financial markets. Cheung, Chou, and Lei (2015) and Lei (2015) have demonstrated that exchange-traded barrier options trade popularly in Hong Kong. Valuing barrier options has been proposed in several studies such as those by Rubinstein and Reiner (1991), Rich (1994), and Hui (1997). These studies have all assumed that the dynamics of the underlying asset prices follow the Black–Scholes (BS) (1973) dynamics. However, empirical asset-return distributions exhibit heavy tails, leptokurtic properties, volatility smile, and volatility clustering (Huang, Peng, Li, & Ke, 2011). As such, accurate valuation is critical for derivatives pricing, especially for barrier options. Therefore, rectifying the underlying distributions to fit empirical distributions has become a well-known, critical issue for pricing barrier options.

Incorporating jump risk into valuation models can capture return-distribution heavy tails and leptokurtic properties (McIntyre & Jackson, 2009; Fan, Luo, & Wu, 2017). Bakshi, Cao, and Chen (1997) indicated that option pricing formulas based on jump-diffusion (JD) representation exhibit less bias. Boyarchenko and Levendorskiĭ (2002) and Levendorskiĭ (2004) have used a Levy model for pricing and hedging interest-rate barrier options. The JD model includes a single jump component for capturing the impact of news arrival on security prices. However, the model cannot distinguish a good news impact from a bad news impact by using its intensity or distributional characteristics. Kou (2002) proposed a double exponential jump-diffusion (DEJD) model in which a single Poisson process with a fixed intensity generates jumps in price, and jump magnitudes are drawn from two independent exponential distributions. In addition, Ramezani & Zeng (2004) illustrated the estimation and empirical assessment of the DEJD model. Their results showed that the

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https://doi.org/10.1016/j.iref.2017.11.003
Received 7 January 2016; Received in revised form 5 November 2017; Accepted 12 November 2017
Available online xxxx
1059-0560/

DEJD model used to fit the S&P 500 index performs more effectively than do log-normal JD and ARCH models. Moreover, Sepp (2004) and Kou & Wang (2004) have shown that the DEJD model can produce nearly analytical solutions to many option pricing problems. For example, Kou, Petrella, & Wang (2005) valued a barrier option under the DEJD model, which has a favorable asymmetric jump property that can capture the overreaction and underreaction of asset returns regarding arrivals of good news and bad news. However, the DEJD model cannot capture the volatility-clustering effect. By contrast, regime-switching (RS) models are useful in econometrics and finance for capturing volatility clustering in asset returns because the RS dynamic allows the model's volatility to change with economic conditions over time. In addition, the RS dynamic provides a natural and convenient means of describing the effect on price series of structure changes in economic conditions, which may be attributed to changes in economic fundamentals or a financial crisis (Gonzalez, Powell, Shi, & Wilson, 2005). Hence, Graziano & Rogers (2006) and Eloe, Liu, and Sun (2009) have valued double barrier options under an RS exponential mean-reverting process. Kudryavtsev (2010) priced barrier options under an RS Levy model. In addition, Lin, Lin, & Wu (2015) examined the RS behavior and the nature of jumps in foreign exchange rates. Lin, Wang, & Tsai (2009) provide empirical evidence that hidden Markov chain can be utilized to better describe the stock return dynamics. Chiang, Li, & Chen (2015) examined JPY/USD and EUR/USD FX rates and provided support for the rationale for the use of a Markov-modulated DEJD (MMDEJD) process for formulations. Their evidence supports the use of an MMDEJD process for formulating an underlying process, which can improve the accurate valuation of option pricing.

Exchange rates and commodity prices have usually exhibited mean-reversion behavior. Purchasing power parity (PPP) in economics implies that real exchange rates may exhibit long-run mean-reverting behavior (Glen, 1992). Indeed, Chen & Jeon (1998) indicated that currency assets exhibit mean-reverting behavior. In addition, the seasonality of commodities leads to mean-reverting behavior in price, such as that exhibited in oil (Bessembinder, Coughenour, & Seguin, 1995). Moreover, Schwartz (1997) suggested the use of an exponential Ornstein-Uhlenbeck process for formulating commodity prices. Hui & Lo (2006) and Wong & Lau (2008) have valued barrier options on an exchange rate that is assumed to follow a mean-reverting lognormal process.

The BS model has undergone several improvements that have incorporated a stochastic interest rate (e.g., Abudy & Izhakian, 2013; Bernard, Courtois, & Quittard-Pinon, 2008). Although valuation models with a constant interest rate are appealing because of their simplicity, they are marred by their failure to consider interest rate risk. However, all of the aforementioned models value the path-dependent derivatives under a Markov-modulated JD model with a constant interest rate. Hui & Lo (2006) and Wong & Lau (2008) have all assumed constant interest rates in their valuation of barrier options. Incorporating interest rate risk is critical for improving the accuracy of valuing options. Therefore, a two-dimensional random variable model is adopted for describing random asset-price and interest-rate processes.

Zvan, Vetzal, & Forsyth (2000) provided the PDE methods for pricing barrier options, and Cheuk & Vorst (1996) proposed a trinomial tree method. These two methods are usually used for pricing path-dependent derivatives. Based on the PDE method, Elliott, Siu, & Chan (2014) provided a semi-closed-form formula for pricing barrier options under the RS BS model. However, their model involves too many complex computations when a stochastic interest rate is incorporated into the valuation. Hence, a change-of-numeraire technique is proposed for simplifying the valuation process of barrier options.

By incorporating stochastic interest rates and mean-reversion behavior into the proposed model, we consider the asset price to follow a mean-reversion MMDEJD process, and the stochastic interest rate is modulated by the Cox–Ingersoll–Ross (CIR, 1985) model of the term structure in which the interest rate is always nonnegative and mean-reverting. The accurate valuation of barrier options is thus enhanced under the mean-reversion MMDEJD-CIR framework.

1.1. *Behavior of Kou's model in bear and bull markets*

The behaviors of the exchange rates, JPY/USD and EUR/USD, and Brent crude oil spot prices are next examined under bull and bear markets. The empirical characteristics of these asset prices exhibit evidence consistent with the economic intuition of Kou's model under bull and bear markets, thereby inspiring us to use the MMDEJD model for pricing and hedging barrier options.

First, the aforementioned three markets are distinguished bull from bear markets using the RS model (Hamilton, 1989). The data of the JPY/USD, EUR/USD, and Brent crude oil spot price (Oil) are collected from August 1, 2008, to July 31, 2016. Fig. 1a–1c illustrate the smoothed probability in the JPY/USD, EUR/USD, and Oil bull markets, respectively.

Fig. 1a–1c shows that the bull and bear markets can be partitioned into two cycles, which are presented in Table 1.

Table 2 reports the JPY/USD, EUR/USD, and Oil parameters in the first cycle that are estimated using the maximization likelihood estimation for Kou's model. In addition, the estimated parameters of the jump part are also obtained under the bull and bear markets.

Table 2 shows that all jump parameters are significant, indicating that all the three markets exhibit up and down jump phenomena. By contrast, the JD model with the only up-jump setting cannot capture these up and down jump phenomena. In addition, the estimated parameters $1/\eta_1$ and $1/\eta_2$ presented in Table 2 represent, respectively, the average up and down jump amplitudes. All the three markets show the result $1/\eta_1 < 1/\eta_2$ in the bear market, and their absolute differences are greater than those in the bull market, which indicates that regime shifts and jumps exhibit an obvious relationship. For example, $1/\eta_2$ is 0.00326 (0.00267) in the JPY/USD market, whereas $1/\eta_1$ is 0.00178 (0.00277) in the bear (bull) market. Specifically, the average down-jump amplitude in the bear market is higher than that in the bull market, whereas the average up-jump amplitude is lower in the bear market than that in the bull market. This result signifies that a panic circumstance can easily occur and the resulting frequency of bad news will be higher in the bear market, thereby leading to a deeper down-jump amplitude. In addition, the impact of good news on asset prices is lower in the bear market than in the bull market. This result supports Kou's (2008) statement that bad news in the bear market tends to have a more severe impact than it does in the bull market. Table 2 also shows that the up-jump probability p is higher in the bull market than in the bear market for the JPY/USD and EUR/USD markets. However, the up-jump probability