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Theory and application of an economic performance measure of risk

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ABSTRACT

Homm and Pigorsch (2012a) use the Aumann and Serrano index to develop a new economic performance measure (EPM), which is well known to have advantages over other measures. In this paper, we extend the theory by constructing a one-sample confidence interval of EPM, and construct confidence intervals for the difference of EPMs for two independent samples. We also derive the asymptotic distribution for EPM and for the difference of two EPMs when the samples are independent. We conduct simulations to show the proposed theory performs well for one and two independent samples. The simulations show that the proposed approach is robust in the dependent case. The theory developed is used to construct both one-sample and two-sample confidence intervals of EPMs for Singapore and USA stock indices.

1. Introduction

Believing that less risk averse economic agents tend to accept riskier gambles, Aumann and Serrano (2008) used the reciprocal of the absolute risk aversion (ARA) of an investor with constant ARA to develop a new economic index of riskness, namely the Aumann and Serrano (AS) index. Thereafter, Homm and Pigorsch (2012a) used the AS index to develop a new economic performance measure (EPM), which can be obtained through dividing the mean of an investment portfolio by the AS index.

The AS index is an objective measurement of riskiness, which is independent of an individual's utility and wealth when he or she participates the gamble. It also possesses several appealing theoretical properties of riskiness, including subadditivity, continuity, and positive homogeneity. Moreover, Homm and Pigorsch (2012b) provided an operational interpretation of the AS index. They showed that the AS index is the reciprocal of the adjustment coefficient from the ruin theory, see, Meilijson (2009). Furthermore, according to Meilijson (2009), the order that the AS index imposes on a set of gambles that have to do with the drawdown of the gambles.[1]

Compared with the Foster and Hart’s (2009) operational index of risk (Hart, 2011), the AS index imposes less restriction on the heaviness of the tails, and thus, it can be directly applied to distributions that are commonly used in modeling financial returns. To be precise, Riedel and Hellmann (2015) have demonstrated that Foster-Hart riskiness index does not exist for many common distributions. In addition, Ehsani and Lien (2015) proved that the minimum Foster-Hart riskiness hedge ratio does not exist. On the contrary, Homm

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and Pigorsch (2012b) obtained a closed form expression for the AS index of normal inverse Gaussian-distributed random variables. Schulze (2014) has also derived the closed-form solutions for several distributions, namely, the Exponential, Erlang, χ^2, Gamma, and Variance-Gamma distribution. Schreiber's (2014) index of riskiness is an index of riskiness defined on relative returns, while the AS index defines on absolute returns.

The EPM has many advantages over other commonly-used risk measures, such as the Sharpe ratio. For example, EPM is strictly monotonic with respect to stochastic dominance (SD), and consistently accounts for the mean, variance and higher moments of the returns distribution. If investment returns follow a normal distribution, the EPM and Sharpe ratio have the same ranking in measuring asset performance. Thus, the EPM generalizes the Sharpe ratio with respect to non-normal distributions.

Confidence intervals are usually regarded as more informative than hypothesis tests since they can provide a range of parameter values that reflect the degree of uncertainty in estimation. The confidence interval construction of the Sharpe ratio, a common performance measure of an investment, has been investigated by many researchers. Jobson and Korkie (1981) proposed a popular tool to test the difference of Sharpe ratios of two investment strategies, where the asymptotic distributions of the estimators of the Sharpe and Treynor performance measures are derived. Memmel (2003) corrected a typographical error in the Jobson-Korkie test, without loss of any statistical properties.

The above tests are not valid when returns have tails that are heavier than the normal distribution, or are time series data. Ledoit and Wolf (2008) applied robust inference methods, suggested constructing studentized time series bootstrap confidence intervals for the difference of Sharpe ratios, and declared the two ratios as different if zero is not contained in the interval. Constructing a confidence interval for an estimator is important for studying the statistical properties. Bartlett (1953) introduced the method to construct asymptotic confidence intervals for an unknown parameter, θ, with higher moments of $\partial L/\partial\theta$, especially when the sample variance is heavily skewed for moderate degrees of freedom.

Ghosh (1979) compared two confidence intervals for the binomial parameter by confidence coefficients, the lengths and Neyman shortness, which were constructed based on the extensions of Clopper-Pearson confidence intervals. Brookmeyer and Crowley (1982) constructed confidence intervals for median survival time. Efron (1987) proposed superior bootstrap confidence intervals for a single parameter in a multi-parameter family. However, to the best of our knowledge, few references focus on the construction of confidence intervals for the economic performance measure with the AS index. The present paper focuses on this issue.

We develop the statistical theory to construct one-sample confidence intervals of EPM. For one-sample confidence intervals, we recommend using three approaches, namely the asymptotic method, percentile bootstrap, and studentized bootstrap methods. The percentile bootstrap approach is the easiest approach, while the studentized bootstrap approach improves performance of the percentile bootstrap approach, and obtains more accurate results. The two bootstrap-based methods are Monte Carlo based inference approaches. van der Vaart (1998) gave a detailed introduction of the asymptotic theory, while Hall (1992), Efron (1979), Chernick (2007), Efron and Tibshirani (1993) provided information on both the percentile bootstrap and studentized bootstrap methods.

We extend the theory further by constructing confidence intervals for the difference of EPMs for two independent samples. For two-sample confidence intervals, we recommend using two methods, namely the asymptotic procedure and method of variance estimates recovery (MOVER). MOVER is a strategy that "recovers" variance estimates from the limits of individual sample parameters, and then forms approximate confidence intervals for functions of the parameters, as proposed by Zou and Donner (2008). Zou, Huang, and Zhang (2009) generalized MOVER, and established confidence limits for a linear function of binomial proportions (for further details on MOVER, see Donner and Zou (2012), Dagan, Poolman, and Siegrist (2010) and Newcombe (2016)). The MOVER method is an excellent and simple tool to construct confidence intervals for two independent samples.

In addition, we derive the asymptotic distribution of EPM, and the difference of two EPMs when the samples are independent. We conduct simulations to show the proposed theory performs well for one and two independent samples. The simulations also show that the proposed approach is robust in the dependent case. We apply the theory to construct both one-sample and two-sample confidence intervals of EPMs for stock indices in Singapore and USA.

The remainder of the paper is organized as follows. In Section 2, we present methods of constructing confidence intervals for EPM with one-sample, including the asymptotic method and bootstrap-based approaches. The asymptotic normality of EPM is also derived. Thereafter, we develop the theory for the construction of two-sample confidence intervals for the difference of two independent EPMs by applying both the asymptotic method and MOVER procedure. In Section 3, we conduct simulations of both the one-sample and two-sample confidence intervals for the difference in two independent EPMs. We also conduct simulations for two-sample confidence intervals for the difference in two dependent EPMs. We illustrate the theory by applying the proposed methods to real data analysis by comparing the performance of the Singapore Stock Market Index and Standard & Poor's Composite 500 Index in Section 4. Section 5 concludes the paper. Proofs of the asymptotic results are given in the Appendix.

2. Theory

Let $\tilde{r}$ be the stochastic return of an investment portfolio, r^f be the deterministic risk-free rate, and $r = \tilde{r} - r^f$ be the excess return. The economic performance measurement (EPM) is defined as (Homm & Pigorsch, 2012a):

$$\theta(r) := \mathrm{EPM}(r) = \frac{E(r)}{\mathrm{AS}(r)} = \frac{E(\tilde{r}) - r^f}{\mathrm{AS}(\tilde{r} - r^f)}, \tag{2.1}$$

where $E(r)$ is the expectation of the excess return and $\mathrm{AS}(r)$, the AS index of riskiness (Aumann & Serrano, 2008) of the excess return, is the positive solution, $s > 0$, to the following equation: