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High resolution measurement of internal full-field displacements and strains using global spectral digital volume correlation

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ABSTRACT

Thanks to its ability to non-destructively access internal strains in materials, Digital Volume Correlation (DVC) is gaining growing interest from both experimental and theoretical mechanics communities. One important issue in the implementation of DVC is the considerable computational costs associated with the huge amount of data, which hinders the applications of the technique, especially for high-resolution displacement and strain measurements. In this paper, we propose an accurate global DVC approach based on a Fourier decomposition for the kinematic basis of the sought displacement field. The approach, referred to as IS-DVC, leads to an algorithm whose computational complexity is not considerably increased by increasing the number of Degrees of Freedom (DOF) of the kinematic basis, thus being computationally efficient using the Fast Fourier Transform (FFT). Artificial experiments have been used to evaluate the uncertainties of IS-DVC at high resolutions. Especially, displacement fields of 3D composites with spherical and non-aligned ellipsoidal particles at small scales were reconstructed. Resulting measurements revealed close similarities in terms of strain heterogeneities throughout the volume with the benchmark strains. Furthermore, it was shown that, in the presence of a discontinuity, the measurement uncertainties are not significantly affected, except for regions surrounding the discontinuity, hence validating the robustness of the reconstructed displacement field at a large number of DOF.

1. Introduction

The Digital Image Correlation (DIC) technique has established itself as an important tool in the area of experimental mechanics for more than three decades [1]. The valuable knowledge on the full-field displacements and strains that the technique provides in 2D has paved the way for interesting applications, such as mechanical properties [2,3] and damage law [4,5] identification, and strain mapping [6–8]. The theoretical framework of DIC can be expanded into three dimensions, in which case it is called Digital Volume Correlation (DVC) [9]. DVC has found emerging applications in the past decade [9–18] concurrently with the advances made in 3D imaging technologies, such as X-ray tomography and confocal microscopy. DVC allows for non-destructively evaluating displacements and strains *inside* materials. However, practical issues related to image acquisition and calculation have not only limited the applications of DVC, but also hindered the full exploitation of DIC's accuracy. Some of these issues are briefly discussed in the sequel:

- *Material limitation*: The choice of material for DVC is often dictated by the limitations of the employed imaging modality in revealing sufficient features for a reliable correlation. For example, in X-ray tomography, these features should stem from the difference between the capacities of the constituent phases in attenuating the emitted X-ray energy. Materials with cellular or granular structures have shown to be suitable for this purpose [9,13,14,17,19]. Metal matrix composites, such as nodular graphite cast iron [18] or man-made composites [20], have also been studied. In the latter, small particles can be added during the elaboration process in order to create the required texture. These particles should be chosen so as not to significantly alter the overall mechanical properties [20].
- *Imaging artifacts*: Spurious features are inevitably introduced into volume images during the acquisition process. Such phenomena lead to perturbations in the measured displacements. Although image pre-processing partly alleviates the problem [21,22], further regularization of the inverse problem might be required for high-resolution DVC [23].
- *Algorithm implementations*: The first developments of DVC were based on local approaches [9,24–26]. Later, global approaches were developed based on trilinear Finite Element (FE) shape

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functions [19], as well as enriched FE shape functions for specimens containing cracks [27]. However, as a result of the extension from 2D to 3D, the amount of data as well as the number of Degrees of Freedom (DOF) for DVC is significantly increased in both local and global approaches, when compared to their 2D counterparts. Consequently, DVC algorithms are highly demanding in terms of computer resources. This limitation has hindered the practical application of high-resolution DVC.

The present paper deals with the numerical aspects of DVC, specifically as far as high-resolution DVC[1] is concerned. We propose a global approach based on Fourier basis functions referred to as Improved Spectral DVC (IS-DVC) hereinafter. Being an extension to 3D of the Improved Spectral Approach (ISA) [29–31], IS-DVC makes use of the Fast Fourier Transform (FFT) to convert the computationally cumbersome system of equations in the Fourier domain to an explicit equation for the displacement field into the spatial domain. The expression thus found can be evaluated quite efficiently. The interesting feature of the approach lies in the fact that the complexity of the correlation procedure does not significantly increase for larger number of DOF.

The theoretical framework of the IS-DVC is explained in Sections 3 and 3.1. The approach is then evaluated on a series of artificial experiments. Simulated experiments requiring high resolution DVC, such as composite materials at micro-scale, were considered, details of which are explained in Section 4. Finally, the results and discussion are presented in Section 5.

2. Background

2.1. Concept of pattern matching

Let $f(\mathbf{x})$ and $g(\mathbf{x})$ represent the intensity functions of spatial coordinates $\mathbf{x}=(x,y,z)$ corresponding to the undeformed and deformed images, respectively. In ideal conditions, these two configurations are correlated through a mapping of coordinates expressed by the following relation:

$$f(\mathbf{x})=g(\check{\mathbf{x}}) \quad \text{where} \tag{1a}$$

$$\check{\mathbf{x}}=\mathbf{x}+\mathbf{u}_{\text{exact}}(\mathbf{x}) \tag{1b}$$

where $\mathbf{u}_{\text{exact}}(\mathbf{x})$ is the displacement vector field resulting from the applied loads. The exact displacement, in the Volume of Interest (VOI), is estimated by a mathematical function with specified Degrees of Freedom (DOF), i.e.

$$\mathbf{u}_{\text{exact}}(\mathbf{x})\approx\mathbf{u}(\mathbf{x};\mathbf{p}) \tag{2}$$

where $\mathbf{p}$ is the set of parameters representing the DOF that should be determined from the pattern matching. The pattern matching consists in finding the parameters that minimize the gap between $f(\mathbf{x})$ and $g(\check{\mathbf{x}})$. This can be expressed as follows:

$$\mathbf{p}_{\mathbf{opt}}=\underset{\mathbf{p}\in\mathcal{A}}{\operatorname{argmin}}\left\{\int_{\text{VOI}}\left[f(\mathbf{x})-g(\mathbf{x}+\mathbf{u}(\mathbf{x};\mathbf{p}))\right]^2\mathbf{dx}\right\} \tag{3}$$

where $\mathcal{A}$ denotes the set of admissible choices for $\mathbf{p}$ and the integrand is called the *squared correlation residuals* that should be minimized. For the sake of simplicity, the above formulation is written assuming that the intensity functions are continuous, hence the use of integral operators. In practice, the discrete image functions are interpolated using proper interpolation methods [32–34] in order to perform the optimization at sub-pixel positions.

[1] Throughout this paper, the resolution of a DVC algorithm refers to that of displacement and strain measurements. In this sense, the resolution is defined as "the smallest change in a quantity being measured that causes a perceptible change in the corresponding indication." (ISO/IEC guide [28]).

2.2. Resolution strategy

Different approaches of DIC depend, in the first place, on how the sought displacement field is formulated. Nevertheless, no matter how it is formulated, the displacement fields for different approaches can be expressed as the linear combination of several chosen basis functions [35], which can be expressed in the form of the following vector product:

$$\mathbf{u}(\mathbf{x};\mathbf{p})=[\psi_1(\mathbf{x})\ \psi_2(\mathbf{x})\ \cdots\ \psi_K(\mathbf{x})]\begin{bmatrix}\boldsymbol{\upsilon}_1\\ \boldsymbol{\upsilon}_2\\ \vdots\\ \boldsymbol{\upsilon}_K\end{bmatrix} \tag{4a}$$

$$\text{where}\quad \mathbf{p}\equiv\{\boldsymbol{\upsilon}_n|n=1,2,\ldots,K\} \tag{4b}$$

$\boldsymbol{\upsilon}_n$ are the sequence of unknown 3×1 (or 2×1 in DIC) vectors associated with basis functions $\psi_n(\mathbf{x})$ and K is the total number of basis functions. Except for some approaches (for example in [26]), a Newton iterative strategy is often used to solve the problem (3). The iterations start with an initial solution $\mathbf{p}^{(0)}$ (leading to $\mathbf{u}^{(0)}$), at iteration i, $g(\mathbf{x}+\mathbf{u})$ is corrected for $\mathbf{u}^{(i-1)}$ and the new solution lies in finding the increment $\delta\mathbf{u}^{(i)}=\mathbf{u}^{(i)}-\mathbf{u}^{(i-1)}$. It is assumed that the sought increment, $\delta\mathbf{u}^{(i)}$, is small enough so that one can linearize $g(\mathbf{x}+\mathbf{u}^{(i-1)}(\mathbf{x})+\delta\mathbf{u}(\mathbf{x}))$ as follows:

$$g(\mathbf{x}+\mathbf{u}^{(i-1)}(\mathbf{x})+\delta\mathbf{u}(\mathbf{x}))\approx g(\mathbf{x}+\mathbf{u}^{(i-1)}(\mathbf{x}))+\nabla_{\mathbf{x}}^{T}g(\mathbf{x}+\mathbf{u}^{(i-1)}(\mathbf{x}))\delta\mathbf{u}(\mathbf{x}) \tag{5a}$$

$$\text{provided that } \forall\mathbf{x},\quad \|\delta\mathbf{u}(\mathbf{x})\|<\|\boldsymbol{\eta}\|. \tag{5b}$$

where $\boldsymbol{\eta}$ is a small real vector, $\nabla_{\mathbf{x}}$ denotes the gradient operator with respect to vector $\mathbf{x}$, and $\square^T$ indicates the vector transpose. Therefore, the First-order Optimality for the problem (3) is written as follows:

$$\nabla_{\boldsymbol{\upsilon}_n}\left[\int_{\text{VOI}}[f(\mathbf{x})-\breve{g}^{(i-1)}(\mathbf{x})-\nabla_{\mathbf{x}}^{T}\breve{g}^{(i-1)}(\mathbf{x})\delta\mathbf{u}(\mathbf{x};\mathbf{p})]^2\,\mathbf{dx}\right]=\mathbf{0},\quad n=1,2,\ldots,K \tag{6}$$

and

$$\breve{g}^{(i)}(\mathbf{x})=g(\mathbf{x}+\mathbf{u}^{(i)}(\mathbf{x})) \tag{7}$$

By applying the differentiation and after simplifications, Eq. (6) is turned into:

$$\int_{\text{VOI}}\left(\nabla_{\mathbf{x}}^{T}\breve{g}^{(i-1)}(\mathbf{x})\begin{bmatrix}\psi_1(\mathbf{x}) & \psi_2(\mathbf{x}) & \cdots & \psi_K(\mathbf{x})\end{bmatrix}\begin{bmatrix}\boldsymbol{\upsilon}_1\\ \boldsymbol{\upsilon}_2\\ \vdots\\ \boldsymbol{\upsilon}_K\end{bmatrix}\psi_n(\mathbf{x})\nabla_{\mathbf{x}}\breve{g}^{(i-1)}(\mathbf{x})\right)\mathbf{dx}$$

$$=\int_{\text{VOI}}((f(\mathbf{x})-\breve{g}^{(i-1)}(\mathbf{x}))\psi_n(\mathbf{x})\nabla_{\mathbf{x}}\breve{g}^{(i-1)}(\mathbf{x}))\,\mathbf{dx},\quad n=1,2,\ldots,K \tag{8}$$

The above equation can be reorganized into a system of linear equations, i.e.

$$\begin{bmatrix}\mathbf{J}_{11} & \mathbf{J}_{12} & \cdots & \mathbf{J}_{1K}\\ \mathbf{J}_{21} & \ddots & & \vdots\\ \vdots & & \ddots & \vdots\\ \mathbf{J}_{K1} & \cdots & \cdots & \mathbf{J}_{KK}\end{bmatrix}\begin{bmatrix}\boldsymbol{\upsilon}_1\\ \boldsymbol{\upsilon}_2\\ \vdots\\ \boldsymbol{\upsilon}_K\end{bmatrix}=\begin{bmatrix}\boldsymbol{\rho}_1\\ \boldsymbol{\rho}_2\\ \vdots\\ \boldsymbol{\rho}_K\end{bmatrix} \tag{9a}$$

where $\mathbf{J}_{mn}$ and $\boldsymbol{\rho}_m$ are 3×3 (or 2×2) and 3×1 (or 2×1) matrices calculated from the following equations:

$$\mathbf{J}_{mn}=\int_{\text{VOI}}((\nabla_{\mathbf{x}}\breve{g}^{(i-1)}\otimes\nabla_{\mathbf{x}}\breve{g}^{(i-1)})(\mathbf{x})\psi_m(\mathbf{x})\psi_n(\mathbf{x}))\,\mathbf{dx} \tag{9b}$$

$$\boldsymbol{\rho}_m=\int_{\text{VOI}}((f(\mathbf{x})-\breve{g}^{(i-1)}(\mathbf{x}))\psi_m(\mathbf{x})\nabla_{\mathbf{x}}\breve{g}^{(i-1)}(\mathbf{x}))\,\mathbf{dx} \tag{9c}$$