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IMPOSING CONCAVITY AND THE NULL-JOINTNESS PROPERTY ON THE PRODUCTION POSSIBILITIES FRONTIER IN CASE OF POLLUTING TECHNOLOGIES

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Economic theory requires the directional distance functions used to study the properties of production possibility sets of polluting technologies to be concave in both outputs, while the implied production possibilities frontier (PPF) is required to be concave with respect to the bad output. However, existing estimation frameworks do not preclude the estimation of convex PPFs. We analyze geometrical properties of the quadratic approximation to the directional output distance functions to derive a constraint that guarantees PPF concavity and consider the issue of imposing the property of null-jointness on the production possibilities set, which is also required by theory. We simulate a dataset corresponding to a concave PPF and show that in case concavity and null-jointness constraints are not imposed, it is possible that the conventional estimation framework may lead to erroneous conclusions with respect to the type of curvature of both the directional output distance function, and the PPF.

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I. Introduction

It is convenient to model polluting production processes in terms of the multi-output technologies with at least one output, e.g., CO_2 emissions, being an undesirable bad. Output distance functions suggested by Shephard (1970) are a useful analytical tool allowing one to quantify such technologies without having to aggregate multiple outputs into a single output index. In its essence, the output distance function is

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representing the distance between observed output combinations, and their efficient projections on the production possibilities frontier (PPF) with greater values of the distance function corresponding to less productive efficiency. Distance to the PPF in this context is measured along a vector emanating from the observed output mix, for example a vector (1,1) corresponding to a simultaneous increase in the amounts of both outputs.

Directional output distance functions developed by Fare et al. (2005) are well-suited for the analysis of polluting technologies since the directional vector along which the efficient projection is computed implies an increase in the good output(s), as is the case with the conventional output distance function, along with the *reduction* in the bad output, which corresponds to the idea of reducing pollution levels. In this study we deal with the two-output polluting technology with one good output y, such as GDP, and one bad output b, such as the CO2 emissions levels.

Given the global importance of reducing pollution levels, computing the costs of such reduction is a necessary task. One of the more popular approaches undertaken in the literature is to exploit the duality between the output distance function and the revenue function to compute the shadow price of an undesirable output as a slope of the PPF at the efficient projection point for each observed combination of outputs, often referred to as the marginal abatement costs (MAC) of reducing the bad output.

Most empirical studies that estimate output distance functions, directional or not, employ a quadratic approximation to the true distance function whose parameters are estimated by minimizing the sum of individual values of the distance function subject to a number of constraints that reflect the desired properties of the underlying production possibilities set (PPS). The translation property constraint, for instance, makes sure that the estimated quadratic function in two outputs actually has the distance function properties, i.e., it turns into zero if the good output is increased, and the bad output is decreased by the value of the approximating quadratic function times the corresponding component of the directional vector. Monotonicity constraints require the (directional) output distance function to increase in the bad output, and to decrease in the good one. If monotonicity constraints hold, the resulting PPF is upward sloping, reflecting the desirable property of the positive MACs.

An important theoretical property of the PPS of the polluting technology is that its production possibilities frontier, viewed as a function $y = f^{PPF}(b)$ of the level of bad output, has to be concave, i.e., $y''(b) \leq 0$, see, e.g.,
Fare et al. (1993). In addition, the distance function should inherit the properties of