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Two-dimensional continuous wavelet transform algorithm for phase extraction of two-step arbitrarily phase-shifted interferograms

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ABSTRACT

By virtue of the anti-noise and anti-defect abilities, in this paper, a two-dimensional continuous wavelet transform algorithm is proposed to analyze two-step arbitrarily phase-shifted interferograms with an orthonormalization process. The novel algorithm takes the advantages of the two existing ones, and it has a remarkable ability to accurately and automatically extract full-field phase distribution from two phase-shifted interferograms where they may contain arbitrary and unknown phase shift, complex fringes with phase ambiguity, large fringe-frequency variations, noise, defects and corrupted fringes. The validity of the algorithm is demonstrated by both computer simulation and real experiments.

1. Introduction

The phase-shifting (PS) technique is commonly used in optical fringe pattern or interferogram analysis because of its many attractive advantages. The drawback of the PS technique is that it is sensitive to noise and it cannot cope with the defects or corrupted fringes existed in the interferograms. This impairs the application of the PS technique to practice where anti-noise and anti-defect processing abilities are desired. When using the PS technique to analyze noisy interferograms, a typical way to reduce the noise level is to apply a filtering process as a pre- or post-processing step [1]. For interferograms with defects or corrupted fringes, special handling and complicated schemes are normally required, such as using a mask map and phasor image processing [2], a quadrature filter [3], and a frequency transfer function [4]. Nevertheless, the noise and defects generally cannot be considerably and simultaneously eliminated with the above schemes, so a more robust anti-noise and anti-defect analysis algorithm is highly demanded. Representative concepts suitable for this purpose include the windowed Fourier transform [5], the advanced phase demodulation [6,7], and the wavelet transform [8–10] approaches.

Recently, a hybrid two-dimensional continuous wavelet transform (2D-CWT) technique, combined with the classical PS technique, has been developed to obtain the full-field phase distribution from interferograms that contain complex fringes, noise, defects and corrupted fringes [11]. The hybrid technique inherits the merits of both the 2D-CWT and the PS techniques; on the other hand, it is inevitably constrained by certain requirements of the two techniques. Particularly, the hybrid technique needs at least three phase-shifted interferograms, in which the phase-shifting amount must be fixed at a specific value and cannot be arbitrarily chosen. It is noteworthy, however, that it is possible to extract phase from arbitrarily phase-shifted interferograms using advanced PS algorithms [12,13] or fulfill the phase analysis with just two phase-shifted interferograms [14–16,18]. Although these algorithms cannot cope well with the noise and fringe defects, they help reveal a possibility that the corresponding concepts could be incorporated into the hybrid 2D-CWT technique to achieve the desired anti-noise and anti-defect phase extraction.

In this paper, a novel 2D-CWT algorithm is proposed to analyze two-step arbitrarily phase-shifted interferograms by integrating the interferogram orthonormalization process and the existing 2D-CWT algorithm. The advanced algorithm can automatically extract full-field phase distribution from two phase-shifted interferograms, no matter whether the phase shift is arbitrary and unknown or not. In addition, the technique does not require pre- or post-processing, and the interferograms may contain complex fringes, large fringe-frequency variations, noise, defects or corrupted fringes.

2. Principle

2.1. Interferogram orthonormalization process

The two-step phase-shifted interferograms can be mathematically expressed as follows:

$$I_1(\mathbf{x}) = I_b(\mathbf{x}) + I_a(\mathbf{x})\cos[\phi(\mathbf{x})] \tag{1}$$

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$$I_2(\mathbf{x}) = I_b(\mathbf{x}) + I_a(\mathbf{x}) \cos[\phi(\mathbf{x}) + \delta] \tag{2}$$

where $\mathbf{x} \in \mathbb{R}^2$ indicates the 2D coordinates of each pixel; I_b, I_a, and ϕ denote the background intensity, the modulation amplitude, and the angular phase, respectively; δ is the phase-shifting amount.

In practice, it is reasonable to assume that I_b can be filtered out by mean intensity subtraction; thus Eqs. (1) and (2) can be written as follows:

$$I_1(\mathbf{x}) = I_a(\mathbf{x}) \cos[\phi(\mathbf{x})] \tag{3}$$

$$I_2(\mathbf{x}) = I_a(\mathbf{x}) \cos[\phi(\mathbf{x}) + \delta] \tag{4}$$

Among the various two-step phase-shifting (TSPS) techniques, the one utilizing the Gram–Schmidt orthonormalization (GSO) algorithm is capable of yielding the highest accuracy while being simple and fast [17]. With the GSO algorithm and treating each interferogram as a vector, the projection operator $\mathcal{P}$ that projects the vector $I_1(\mathbf{x})$ orthogonally onto the line spanned by the vector $I_2(\mathbf{x})$ can be defined as follows:

$$\mathcal{P}_{I_1(\mathbf{x})}[I_2(\mathbf{x})] = \frac{\langle I_2(\mathbf{x}), I_1(\mathbf{x})\rangle}{\langle I_1(\mathbf{x}), I_1(\mathbf{x})\rangle} I_1(\mathbf{x}) \tag{5}$$

where $\langle\rangle$ represents the inner product of two vectors. Substituting Eqs. (3) and (4) into Eq. (5) yields

$$\mathcal{P}_{I_1(\mathbf{x})}[I_2(\mathbf{x})] = \frac{\sum I_a^2(\mathbf{x}) \cos[\phi(\mathbf{x})] \cos[\phi(\mathbf{x}) + \delta]}{\sum I_a^2(\mathbf{x}) \cos^2[\phi(\mathbf{x})]} I_a(\mathbf{x}) \cos[\phi(\mathbf{x})] \tag{6}$$

where $\sum$ indicates the summation over all of the pixels in each image. If the total fringe number is larger than one in the interferograms, it can be derived that $|\sum \cos^2[\phi(\mathbf{x})] \cos(\delta)| \ll |\sum \cos[\phi(\mathbf{x})] \sin[\phi(\mathbf{x})] \sin(\delta)|$ [17]. Based on this relation, we can have

$$\sum \cos[\phi(\mathbf{x})] \cos[\phi(\mathbf{x}) + \delta] \approx \sum \cos^2[\phi(\mathbf{x})] \cos(\delta) \tag{7}$$

Next, since the fringe modulation amplitude $I_a(\mathbf{x})$ usually does not have large variations in an interferogram, Eq. (6) can be simplified as follows:

$$\begin{aligned}\mathcal{P}_{I_1(\mathbf{x})}[I_2(\mathbf{x})] &\approx \frac{\sum I_a^2(\mathbf{x}) \cos^2[\phi(\mathbf{x})] \cos(\delta)}{\sum I_a^2(\mathbf{x}) \cos^2[\phi(\mathbf{x})]} I_a(\mathbf{x}) \cos[\phi(\mathbf{x})] \\ &= I_a(\mathbf{x}) \cos[\phi(\mathbf{x})] \cos(\delta)\end{aligned} \tag{8}$$

Then the vector which is orthogonal to $I_1(\mathbf{x})$ can be obtained by the following:

$$I_\perp(\mathbf{x}) = I_2(\mathbf{x}) - \mathcal{P}_{I_1(\mathbf{x})}[I_2(\mathbf{x})] \tag{9}$$

Substituting Eqs. (4) and (8) into (9) yields

$$\begin{aligned}I_\perp(\mathbf{x}) &= I_a(\mathbf{x}) \cos[\phi(\mathbf{x}) + \delta] - I_a(\mathbf{x}) \cos[\phi(\mathbf{x})] \cos(\delta) \\ &= -I_a(\mathbf{x}) \sin[\phi(\mathbf{x})] \sin(\delta)\end{aligned} \tag{10}$$

Normalizing $I_1(\mathbf{x})$ and $I_\perp(\mathbf{x})$ gives

$$\widehat{I}_1(\mathbf{x}) = \frac{I_1(\mathbf{x})}{\|I_1(\mathbf{x})\|} = \frac{I_a(\mathbf{x}) \cos[\phi(\mathbf{x})]}{\sqrt{\sum I_a^2(\mathbf{x}) \cos^2[\phi(\mathbf{x})]}} \tag{11}$$

$$\widehat{I}_\perp(\mathbf{x}) = \frac{I_\perp(\mathbf{x})}{\|I_\perp(\mathbf{x})\|} = \frac{-I_a(\mathbf{x}) \sin[\phi(\mathbf{x})]}{\sqrt{\sum I_a^2(\mathbf{x}) \sin^2[\phi(\mathbf{x})]}} \tag{12}$$

Again, if there are more than one fringe in the interferograms and the fringe modulation amplitude does not have large variations across the interferograms, it can be seen that $\|I_1(\mathbf{x})\| \cong \|I_\perp(\mathbf{x})\|$ [17]. Consequently, the phase distribution of the two-step phase-shifted interferograms can be obtained as follows:

$$\phi(\mathbf{x}) = \tan^{-1}\left[-\frac{\widehat{I}_\perp(\mathbf{x})}{\widehat{I}_1(\mathbf{x})}\right] \tag{13}$$

2.2. 2D-CWT technique

Eq. (13) for the TSPS algorithm is based on the aforementioned assumption that $I_b(\mathbf{x})$ and $I_a(\mathbf{x})$ are uniform and constant across each entire image. This condition is not easy to satisfy in real applications, especially when noise and defects exist. Unlike the PS technique, the 2D-CWT technique analyzes the fringes in each small local region where $I_b(\mathbf{x})$ and $I_a(\mathbf{x})$ can be more reasonably treated as uniform. The analysis is technically equivalent to finding a local fringe pattern that matches with the real fringe pattern, which can help eliminate the noise and help recover the fringe information at the defect locations where fringes are corrupted. Because of this intrinsic advantage, the 2D-CWT technique should be technically suitable for analyzing two-step phase-shifted interferograms in practice, as elaborated below.

With the two-step phase-shifted interferograms, an analytic interferogram can be constructed as follows:

$$I(\mathbf{x}) = \widehat{I}_1(\mathbf{x}) - j\widehat{I}_\perp(\mathbf{x}) \tag{14}$$

where j is the imaginary unit. This analytic interferogram can be analyzed by employing the 2D-CWT technique with the direct ridge detection algorithm [11]. The 2D-CWT of the analytic interferogram is defined as follows:

$$\mathcal{W}(\mathbf{u}, s, \theta) \equiv \langle I, \psi_{\mathbf{u},s,\theta}\rangle = \int_{\mathbb{R}^2} I(\mathbf{x}) \psi^*_{\mathbf{u},s,\theta}(\mathbf{x})\, \mathrm{d}^2\mathbf{x} \tag{15}$$

where $\mathcal{W}$ is the wavelet coefficient, $\mathbf{u} \in \mathbb{R}^2$ denotes pixel position, $s \in \mathbb{R}_+$ is a scale factor, $\theta \in [0, 2\pi)$ is a rotation angle, I indicates the analytic interferogram, $*$ represents the complex conjugate operator, and $\psi_{\mathbf{u},s,\theta}$ is the wavelet function defined by the following:

$$\psi_{\mathbf{u},s,\theta}(\mathbf{x}) = \psi[s^{-1} r_{-\theta}(\mathbf{x} - \mathbf{u})] \tag{16}$$

where $r_{-\theta}$ is the conventional 2×2 rotation matrix corresponding to θ.

A series of wavelet coefficients at each pixel location can be obtained from Eq. (15) using various parameter pairs (s, θ) [19]. The case where the wavelet coefficient has the maximum magnitude is called the wavelet ridge. This can be expressed as follows:

$$\mathcal{W}(\mathbf{u})_{\text{ridge}} = \mathcal{W}(\mathbf{u}, s_{\text{ridge}}, \theta_{\text{ridge}}) \tag{17}$$

where

$$(s_{\text{ridge}}, \theta_{\text{ridge}}) = \arg \max_{\substack{s \in \mathbb{R}^+ \\ \theta \in [0, 2\pi)}} \{|\mathcal{W}(\mathbf{u}, s, \theta)|\} \tag{18}$$

The phase at $\mathbf{u}$ can then be calculated from

$$\phi(\mathbf{u}) = \tan^{-1}\left\{\frac{\Im[\mathcal{W}(\mathbf{u})_{\text{ridge}}]}{\Re[\mathcal{W}(\mathbf{u})_{\text{ridge}}]}\right\} \tag{19}$$

where $\Im$ and $\Re$ denote imaginary and real parts of a complex value, respectively [20].

2.3. Procedure

Considering that the convolution involved in the 2D-CWT, i.e., Eq. (15), can be implemented by using fast Fourier transform (FFT), the procedure of the proposed 2D-CWT algorithm previously described is summarized as follows:

1. Project $I_2(\mathbf{x})$ orthogonally onto $I_1(\mathbf{x})$ by using Eq. (5), and obtain the orthogonal signal $I_\perp(\mathbf{x})$ through using Eq. (9).
2. Normalize $I_1(\mathbf{x})$ and $I_\perp(\mathbf{x})$ as $\widehat{I}_1(\mathbf{x})$ and $\widehat{I}_\perp(\mathbf{x})$, respectively.
3. Construct the analytic signal $I(\mathbf{x})$ from the orthonormalized interferograms $\widehat{I}_1(\mathbf{x})$ and $\widehat{I}_\perp$ with Eq. (14), and calculate the FFT of $I(\mathbf{x})$ as $\tilde{I}(\boldsymbol{\omega})$.