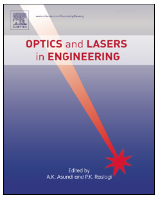
Optics and Lasers in Engineering 55 (2014) 226–231

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng



Phase-visibility modulating interferometry by binary non-quadrature amplitude modulation with neutral density filters

Uriel Rivera-Ortega, Cruz Meneses-Fabian*, Gustavo Rodriguez-Zurita, Carlos Robledo-Sanchez

Facultad de Ciencias Físico-Matemáticas, Benemérita Universidad Autónoma de Puebla, Apdo. Postal 165, Puebla, Pue., México

ARTICLE INFO

Article history:
Received 24 August 2013
Received in revised form
1 November 2013
Accepted 7 November 2013
Available online 11 December 2013

Keywords:
Interferometry
Phase retrieval
Phase-visibility modulating
Amplitude modulation

ABSTRACT

An alternative method for phase retrieval based on spatial and binary non-quadrature amplitude modulation (NQAM) is presented. This proposal is based on the superposition of a probe beam with a reference beam modulated in phase and amplitude (PAM) by NQAM, which is implemented by two neutral density filters (NDF) in a three-beam Mach–Zehnder interferometer (MZI). The principal advantage of this proposal lies in an analytical relationship between the variations of phase and visibility in an interferogram with the variations in the amplitudes of the reference beams used to implement NQAM; thus, the interferograms can be normalized and their introduced phase variations can be known from the measured intensities. Consequently it is possible to successfully retrieve the object phase. It is worthy to note that this method is capable of accepting that the phase and visibility variations in the interferograms could be spatial functions.

1. Introduction

Many experimental methods and algorithms for phase retrieval in two-beam interferometry have been proposed and extensively developed [1]. Phase-shifting interferometry (PSI) is one of the most widely used methods [2,3], which consists in generating several known phase-steps between two beams in order to form a resoluble equations system [4,5]. However a very well described and calibrated phase shifter with high accuracy has to be employed, which needs hardware and software with special characteristics, resulting in a high cost and additional work. To overcome this inconvenience, methods for phase retrieval when the phase shifter is miscalibrated known as generalized phase-shifting interferometry (GPSI) have been developed, [6–11], but a phase-step without change of visibility must be generated. Experimentally, a phase-step has been introduced by different principles and ways, for instance: by changing the optical path by displacing a mirror with a piezoelectric transducer [12], by altering the refractive index by means of tilting a glass plate [13], by changing the frequency between the two beams by means of the Zeeman effect [14] or the wavelength with the Doppler effect [15], by polarization modulation [16], or by displacing a grating [17], or recently, by QAM and NQAM in phase modulation mode [18,19], among others. On the other hand, carrier fringes interferometry (CFI) is another widely used method for phase retrieval [20]. CFI is based on introducing a linear phase term into the interferogram in order to separate the object phase information in the Fourier domain, filtering and missing the carrier frequency, then returning to spatial domain for phase retrieval. Typically a carrier frequency can be introduced by tilting a mirror [21], by using a wedge prism [22], or by placing a grating outside the Fourier plane in a double-aperture common-path interferometer [23]; however these changers of linear phase need to be carefully calibrated. Studies for when a quadratic phase term is introduced also have been developed [5,24].

From another point of view, typically in PSI, GPSI or CFI the reference beam is modulated in phase only (PM), in order to obtain a constant visibility and phase-steps and linear or quadratic shifts in the interferograms. To our knowledge, there is no method for phase retrieval when the order of the introduced phase is superior than the second order and when the visibility is also changed, which occurs when the reference beam is modulated in PAM mode.

In this manuscript, we introduce a new method which is able to retrieve the object phase even when the introduced phases and visibilities are spatial functions, and different between each interferogram, which are obtained when a PAM in the reference beam is carried out by NQAM. The methods mentioned above such as PSI, GPSI and CFI are unable to resolve these types of interferograms. The relationship between the amplitude modulation (AM) of the beams for the implementation of NQAM and the introduced phase and visibilities allows the normalization of the interferograms from the measured intensities, and for this reason the phase retrieval can be successfully achieved. NQAM is implemented in two arms of a three-beam Mach–Zehnder interferometer, where their AM is carried out

---

* Corresponding author.
E-mail address: cmeneses@fcfm.buap.mx (C. Meneses-Fabian).

0143-8166/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.optlaseng.2013.11.007

by two NDF to be implemented in PAM mode; therefore the introduced phases and visibilities vary spatially in the interferograms.

2. Theoretical model

The schema depicted in Fig. 1 shows a three-beam interferometer built with two Mach–Zehnder connected in series. The beamsplitter BS_1 divides in two parts the beam leaving from the laser; one part crosses a neutral density filter NDF_{r1} in order to modulate it in amplitude E_{r1}. This beam is named the reference beam $r1$. The second part is divided by another beamsplitter BS_2, where one of the divided beams crosses another neutral density filter NDF_{r2} in order to obtain the second reference beam, $r2$, modulated in amplitude E_{r2}. The other divided beam is expanded, filtered, and collimated in order to cross a phase object under test. This second beam is named as probe beam E_p. The reference beams of amplitudes E_{r1} and E_{r2} are added with BS_5 and the resulting beam, E_r, is immediately expanded, filtered and collimated. With this, the reference beam E_r is obtained in PAM mode by NQAM method. Finally, the sum of E_r and E_p is carried out by BS_6. Note that BS_3 and BS_4 could be mirrors, but we have chosen them as beam splitters to compensate the crossing of different optical path lengths due to the thickness of the rest of the beamsplitters and for the eventual loss of amplitude. Thus, each beam crosses four beam splitters and the amplitude variations are only carried out by the NDF.

Each field in the sum leaving from BS_6 is considered to be coherent, monochromatic, linearly polarized, and traveling on z-direction as a plane wave

$$E_k(x,y,z,t) = A_k(x,y,z)\exp\{i[k_z z - \omega t + \phi_k(x,y,z)]\}, \tag{1}$$

where $k = r1, r2, p$; $k = r1, r2$ denote the reference fields and $k = p$ denotes the field that crosses the phase object, called the probe field. A_k represents the spatial variations of the amplitude because of the neutral density filters, but also because of the non-uniformity of illumination, nonlinearly of the detector, or some defect in the optical components; ϕ_k indicates the phase variations because of optical-path, the optical system's aberrations, changes in the temperature, atmospheric turbulence, mechanical vibrations, and objects under test among others; $k_z = 2\pi/\lambda$ is the wavenumber, with λ representing the wavelength of light, and $i = \sqrt{-1}$ is the imaginary unit.

Let us first consider the sum $E_r = E_{r1} + E_{r2}$, whose description by using Eq. (1), and omitting coordinates (x, y), can be written as

$$E_r(z,t) = [A_{r1}\exp(i\phi_{r1}) + A_{r2}\exp(\phi_{r2})]\exp[i(k_z z - \omega t)] = A_r\exp(i\phi_r)\exp[i(k_z z - \omega t)], \tag{2}$$

where the new optical field, the reference field, has the same oscillation frequency ω and propagation direction k_z, but has a new amplitude A_r and a new initial phase ϕ_r. From the phasor diagram in Fig. 2, it can be easily established that

$$A_r^2(A_{r1}, A_{r2}) = A_{r1}^2 + A_{r2}^2 + 2A_{r1}A_{r2}\cos\Delta\phi_{21}, \tag{3a}$$

$$\tan\Delta\phi_r(A_{r1}, A_{r2}) = \frac{A_{r2}\sin\Delta\phi_{21}}{A_{r1} + A_{r2}\cos\Delta\phi_{21}}, \tag{3b}$$

where $\Delta\phi_{21} = \phi_{r2} - \phi_{r1}$ and $\Delta\phi_{r1} = \phi_r - \phi_{r1}$. Assuming $\Delta\phi_{21}$ temporally constant, from Eq. (3) it can be noted that both A_r and $\Delta\phi_{r1}$ are depending on the amplitudes A_{r1} and A_{r2}, which can be respectively varied by NDF_{r1} and NDF_{r2}; besides, they can be experimentally obtained from the measured intensities by means of $I_{r1} = A_{r1}^2$, $I_{r2} = A_{r2}^2$, and $I_r = A_r^2$. Thus $\Delta\phi_{21}$ can be calculated as

$$\cos\Delta\phi_{21} = \frac{I_r - I_{r1} - I_{r2}}{2\sqrt{I_{r1}I_{r2}}}, \text{ with } I_{r1}, I_{r2} \neq 0 \tag{4}$$

and with $\Delta\phi_{21}$ known from Eq. (4), $\Delta\phi_{r1}$ can be computed from Eq. (3b).

Eqs. (2)–(4) describe in general the case of PAM of the reference field produced by spatial NQAM, which will be implemented in this paper in its digital version (binary) for phase retrieval.

Now, let us consider the total optical field E leaving from the BS_6, given by the sum of three fields $E_{r1} + E_{r2} + E_p$, or, equivalently interpreted as the sum of the two fields $E_r + E_p$. By using Eqs. (1) and (2), the intensity stored by a detector can be modeled as,

$$I = A_r^2 + A_p^2 + 2A_rA_p\cos(\phi - \Delta\phi_{r1}), \tag{5a}$$

which is an interferogram with an additional phase that can be known from Eqs. (3b) and (4), and a visibility given by

$$V = \frac{2A_rA_p}{A_r^2 + A_p^2}, \tag{5b}$$

where $\phi = \phi_p - \phi_{r1}$ represents the phase object as it is typically interpreted in a two-beam interferometer. However ϕ_p can be separated from ϕ if two measurements are done, one of them in

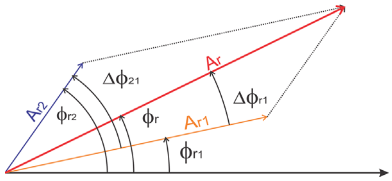


Fig. 2. Phasor diagram of non-quadrature amplitude modulation method.

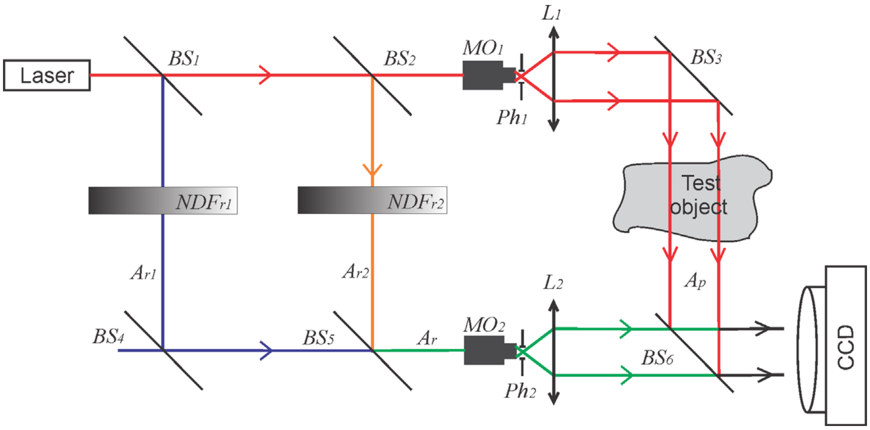


Fig. 1. Three-beam Mach–Zehnder interferometer: BS, Beam splitters; NDF, Neutral density filters; A amplitudes; MO, microscope objective; Ph, Pinhole; L, Convergence lenses; E, Optical beams; CCD, camera.