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Expected Shortfall, spectral risk measures, and the aggravating effect of background risk, or: risk vulnerability and the problem of subadditivity

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ABSTRACT

We analyze spectral risk measures (SRMs) including its most popular representative, Expected Shortfall (ES), with respect to Gollier and Pratt (1996)'s concept of risk vulnerability. We find that SRMs and risk vulnerability are mutually exclusive, owing to the property of subadditivity: while *subadditivity* is commonly regarded as *the* axiomatic cornerstone of SRMs, risk vulnerability, by contrast, prevails if and only if *superadditivity* holds. The lack of risk vulnerability yields questionable predictions in portfolio problems: SRM-decision makers who split their wealth between a risk free and a risky asset do constantly opt for an increase in the risky investment when their deterministic background wealth is complemented by some additional background risk. The more general setting where background wealth is already random and then becomes more risky is not as clear-cut: Any SRM-decision maker may both increase or decrease the risky investment, depending on the concrete instance of the portfolio problem. However, when random background wealth and the risky asset are jointly normally distributed, SRM-decision makers will again unambiguously increase their risky investment. We further conduct a data analysis and discuss possible implications of the findings for regulatory risk management.

1. Introduction

Acerbi (2002) has introduced spectral risk measures (SRMs) as a natural extension to popular Expected Shortfall (ES), and has shown that they are the class of all risk measures satisfying a set of reasonable properties (or axioms) for the purpose of quantifying solvency capital requirements in bank and insurance regulation. Among these properties, subadditivity is of particular relevance: it ensures that SRMs, unlike Value-at-Risk, consistently take into account the positive effects of diversification (e.g., Szegoe (2002)).

Largely inspired by the appealing argument of subadditivity and diversification, the scope of application of ES and SRMs has widened, and they meanwhile also act as a modern counterpart to traditional expected utility (EU)-theory for decision making under risk, as, for example, in portfolio selection (e.g., Adam et al. (2008), Brandtner (2013)), (re-)insurance theory (e.g., Cai et al. (2008), Brandtner and Kürsten (2014)), and operations management (e.g., Jammernegg and Kischka (2007)). However, we still do not fully know whether or under which conditions SRMs are able to provide reasonable predictions within these new fields of application. For example, it is an open question how optimal decisions under SRMs are affected by increasing risk aversion of decision makers and increasing risk – in particular, increasing background risk – in the decision situation at hand, respectively. In EU-theory, these kind of comparative static analyses have been addressed in hundreds of publications, and the results allow restricting the classes of available utility functions to those that are consistent with fundamental economic intuition (for reviews see, e.g., Eeckhoudt and Gollier (2000) and Gollier et al. (2013)). Likewise, these analyses constitute a necessary prerequisite prior to applying SRMs to real-world decision making in order (i) to unveil in which decision situations SRMs are generally able or unable to provide reasonable economic results, and (ii) to further refine the class of SRMs by determining (and ruling out) proper (and improper) subclasses.

A first attempt in this direction has recently been made by Brandtner and Kürsten (2015), who have analyzed SRMs with respect to increasing risk aversion following Arrow (1965) and Pratt (1964), and Ross (1981), respectively. Among others, the results show that in portfolio selection, SRM-decision makers may counterintuitively increase risky investment with increasing risk aversion, and that this effect is especially pronounced under ES. A complementary fundamental problem is how optimal decisions under SRMs are affected by the presence and, in particular, by changes in

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background risk. This question is relevant to both individual and institutional SRM-decision makers (e.g., banks regulated by means of ES, see Section 5 below): while the former regularly face fluctuating background risks such as labor income or real estate, the latter are confronted with non-tradable problem loans or non-current assets held for strategic purposes. As of yet, however, there is no rigorous analysis on whether decisions under SRMs fit with economic intuition when background risk is changing. In this paper, we will argue that fundamental doubts on this fit may be justified, with far-reaching and undesired consequences for decision making and risk management under SRMs both on an individual and an institutional level. Specifically, we will show that ES and SRMs induce adverse risk shifting-incentives in individual and institutional asset portfolios when background risks are explicitly taken into account.

To this end, we adopt the well-established concept of risk vulnerability as proposed by Gollier and Pratt (1996). These scholars were the first to rigorously analyze the effect of background risks on optimal decision making, initially in the context of traditional EU-theory. A decision maker is risk vulnerable if adding an unfair background risk to wealth makes her behave in a more risk-averse way with respect to any other independent risk. Among others, risk vulnerability plays an important role in portfolio problems: Consider a decision maker who splits her wealth between a risk free and a risky asset, initially in the presence of some deterministic background wealth. Now assume that a background risk is added to this initial background wealth. Gollier and Pratt (1996) show that, in line with economic intuition, an EU-decision maker will unambiguously react with an increase in the risk free investment if and only if she is risk vulnerable. Beyond this normative approach, risk vulnerability has also proven to be highly relevant from an experimental point of view. For example, Beaud and Willinger (2015) have recently shown that more than 80% of investors actually reduce the risky investment when being exposed to background risk (see their Section 2 for further experimental evidence).

In this paper, the concept of risk vulnerability is for the first time applied to ES and SRMs. Our contribution is threefold: First, we show that SRMs and risk vulnerability are mutually exclusive, owing to the opposing role of subadditivity in the initial context of bank regulation on the one hand, and in the new context of decision making under risk on the other hand. Within the field of bank regulation, subadditivity is commonly argued to be *the* axiomatic cornerstone of SRMs that ensures that the positive effects of diversification are consistently taken into account when determining solvency capital requirements (e.g., Acerbi (2002), Szegoe (2002)). By contrast, when decision behaviour under risk is modeled using SRMs, it is the same property of subadditivity that prevents risk vulnerability. As a consequence, SRM-decision makers do never act risk vulnerable.

Second, we study the implications of the lack of risk vulnerability for the aforementioned portfolio problem. We show that SRM-decision makers will unambiguously opt for an increase in the risky investment when their deterministic background wealth is complemented by some additional background risk. The more general setting where background wealth is already random and then becomes more risky is not as clear-cut: Any SRM-decision maker may both increase or decrease the risky investment, depending on the concrete instance of the portfolio problem. However, when random background wealth and the risky asset are jointly normally distributed, SRM-decision makers will again unambiguously increase their risky investment. We conduct a data analysis and show that this risk-shifting effect of background risk is economically highly relevant: doubling the background risk yields a reallocation of up to 30% of the risk free investment toward the risky investment.

Third, we discuss possible implications of the findings for the initial SRM-context of bank regulation and argue that the lack of risk vulnerability might question the use of ES – toward which regulators have recently been moving – in regulatory risk management.

The paper proceeds as follows. Section 2 provides the representation and relevant properties of SRMs. Section 3 briefly recalls the concept of risk vulnerability and shows that SRM-decision makers are not risk vulnerable. Sections 4 and 5 address the implications of the lack of risk vulnerability for portfolio selection and regulatory risk management, respectively. Section 6 concludes.

2. Spectral risk measures: representation and properties

Consider the set $\mathcal{X}$ of all measurable, real valued (P&L) random variables X with finite mean, $E(X) < \infty$. The cumulative distribution function and the quantile function of X are given by $F_X(x) = F(x) = P(X \leq x)$ and $F_X^{-1}(p) = F^{-1}(p) = \sup\{x \in \mathbb{R} | F(x) < p\}$, $p \in (0, 1]$, $F^{-1}(0) = \operatorname{ess\,inf}\{X\}$, respectively.

We start with the representation of SRMs (Acerbi (2002)).

Definition 2.1. A mapping $\rho_\phi : \mathcal{X} \to \mathbb{R}$ is called spectral risk measure (SRM) if it is given by

$$\rho_\phi(X) = -\int_0^1 F^{-1}(p) \cdot \phi(p) \mathrm{d}p, \tag{1}$$

where the so-called risk spectrum $\phi : [0, 1] \to \mathbb{R}_0^+$ is a non-increasing density function on (0,1], and the antiderivative $\Phi(p) = \int_0^p \phi(t) \mathrm{d}t$ is continuous on [0, 1].

The latter rather technical condition ensures that the integral in (1) is well-defined and finite. SRMs are linear in the outcomes of a random variable, $x = F^{-1}(p)$, but nonlinear in the probabilities, p, which are "distorted" by means of the risk spectrum ϕ. Technically, SRMs can be seen as a natural counterpart to the EU-functional, which is linear in the probabilities, while the outcomes are distorted by the utility function. SRMs initially have been introduced for the purpose of quantifying solvency capital requirements in bank and insurance regulation, and thus are constructed to satisfy a set of reasonable properties (see, e.g., Acerbi (2004) for a thorough discussion). Among these properties, the following will become particularly relevant below:

1. Linearity: $\rho_\phi(\lambda \cdot X + c) = \lambda \cdot \rho_\phi(X) - c$, for all $X \in \mathcal{X}, \lambda \geq 0, c \in \mathbb{R}$,
2. Subadditivity: $\rho_\phi(X + Y) \leq \rho_\phi(X) + \rho_\phi(Y)$, for all $X, Y \in \mathcal{X}$.

Especially, subadditivity is seen as a cornerstone in modern regulatory risk measurement, as it ensures that SRMs adequately account for the positive effects of diversification. As another relevant property, spectral risk measures are consistent with second order stochastic dominance (SSD): it holds that

$$\int_{-\infty}^t F_X(x)\mathrm{d}x \leq \int_{-\infty}^t F_Y(x)\mathrm{d}x \ \forall t \in \mathbb{R} \Leftrightarrow$$
$$\text{for any SRM } \rho_\phi, \text{ we have } \rho_\phi(X) \leq \rho_\phi(Y). \tag{2}$$

(see Hadar and Russel (1969), Rothschild and Stiglitz (1970) for the concept of SSD, and Leitner (2005), Adam et al. (2008) for the case of SRMs). SRMs, like EU-theory with a concave utility function u, reject SSD-increases in risk by assigning a higher spectral risk to the SSD-riskier random variable Y. Conversely, if a random variable Y is rejected over a random variable X by any SRM, X is riskier than Y in the sense of SSD. Accordingly, an SSD-increase in risk is a meaningful approach to modeling increasing risk under SRMs (as it has been proven to be under traditional EU-theory).