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Point process models for extreme returns: Harnessing implied volatility

R. Herrera[a], A.E. Clements[b,*]

[a] *Facultad de Economía y Negocios, Universidad de Talca, Chile*
[b] *School of Economics and Finance, Queensland University of Technology, Australia*

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ABSTRACT

Forecasting the risk of extreme losses is an important issue in the management of financial risk. There has been a great deal of research examining how option implied volatilities (IV) can be used to forecast asset return volatility. However, the role of IV in the context of predicting extreme risk has received relatively little attention. The potential benefit of IV in forecasting extreme risk is considered within a range of models beginning with the traditional GARCH based approach, along with a number of novel point process models. Univariate models where IV is included as an exogenous variable are considered along with a novel bivariate approach where extreme movements in IV are treated as another point process. It is found that in the context of forecasting Value-at-Risk, the bivariate models produce the most accurate forecasts across a wide range of scenarios.

1. Introduction

Modeling and forecasting extreme losses is an important issue in the management of financial risk meaning that accurate estimates of risk measures such as Value-at-Risk (VaR) have attracted a great deal of research attention. A successful model for dealing with these extreme loss events must capture their tendency to cluster in time.

A number of approaches to deal with the clustering of events have been proposed. McNeil and Frey (2000) develop a two stage method where GARCH models are first applied to model the general time variation in volatility with extreme value theory (EVT) techniques then applied to the residuals. Chavez-Demoulin et al. (2005) propose a novel Peaks Over Threshold (POT) approach for modelling extreme events. To deal with event clustering they employ a self-exciting marked point process, specifically a Hawkes-POT process, where the intensity of the occurrence of extreme events depends on the past events and their associated size or marks. Herrera and Schipp (2013) extend the Hawkes-POT framework of Chavez-Demoulin et al. (2005) in proposing a duration based model to capture the clustering in extreme loss events.

While they have not been considered in this specific context, option implied volatilities (IV) have been widely used for forecasting volatility. As the volatility of the returns on the underlying asset price is an input into option pricing models, an expectation (risk neutral) of volatility is required when pricing options. While IV is a risk neutral estimate, it is well known that IV indices are negatively correlated with the level of stock market indices and are an important measure of short-term expected risk (see, Bekaert and Wu, 2000; Wagner and Szimayer, 2004; Giot, 2005; Becker et al., 2009; Lin and Chang, 2010; Bekaert and Hoerova, 2014, among others), and have been found to be a useful forecast of physical spot volatility in many studies, see Poon and Granger (2003). Blair et al. (2001) find the inclusion of IV as an exogenous variable in GARCH models to be beneficial in terms of forecasting. While not focusing on forecasting, Becker et al. (2009) show that IV contains useful information about future jump activity in returns, which is likely to reflect extreme movements in prices.

Very few studies have focused on the complex extremal dependence between IV and equity returns. Aboura and Wagner (2016) investigate the asymmetric relationship between daily S&P 500 index returns and VIX index changes revealing a contemporaneous volatility-return tail dependence for negative extreme events though not for positive returns. Peng and Ng (2012) analyse the cross-market dependence between five of the most important

* Corresponding author.
E-mail address: a.clements@qut.edu.au (A.E. Clements).

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equity markets and their corresponding volatility indices, finding evidence of asymmetric tail dependence. Hilal et al. (2011) propose a conditional approach for capturing extremal dependence between daily returns on VIX futures and the S&P500. Their empirical analysis shows that VIX futures returns are very sensitive to stock market downside risk.

In this paper, the analysis moves beyond the role of IV in forecasting total volatility to focus on the link to extreme losses and addresses two main questions.

1. How are extreme shocks in an IV index and extreme events in its respective stock market return related?
2. Can this relationship be harnessed to provide superior forecasts of extreme returns?

To address these issues, an approach utilising IV within intensity based point process models for extreme returns is proposed. The first model treats IV as an exogenous variable influencing the intensity and the size distribution of extreme events. A novel alternative view is also proposed based on a bivariate Hawkes-POT model. Extreme movements in IV are treated as events themselves, with their impact on extreme events in equity returns captured through a bivariate Hawkes-POT model. Performance of the proposed methods will be analysed in the context of forecasting extreme losses within a VaR framework. The benchmark approach follows both the earlier forecasting literature in that IV is used as an exogenous variable within the GARCH-EVT framework and the bad environments, good environment (BEGE) model of Bekaert et al. (2015).

An empirical analysis is undertaken where forecasts of the risk of extreme returns are generated for five major equity market indices using their associated IV indices. These forecasts are based on GARCH-EVT, BEGE, univariate and bivariate Hawkes-POT models, and take the form of VaR estimates at a range of levels of significance. It is found that GARCH based forecasts which include IV are often inaccurate. Univariate Hawkes-POT and BEGE models where IV is treated as an exogenous variable outperform the GARCH forecasts, though their forecasts do fail a number of tests for VaR adequacy. The bivariate Hawkes-POT models, where the timing of past extreme increases in IV are treated as a point process, lead to the most accurate forecasts of extreme risk in the widest set of scenarios. The results of this paper show that while IV is beneficial for forecasting extreme risk in equity returns, the framework within which it is used is important. The superior approach is to treat extreme increases in IV as a point process within a bivariate model for extreme returns.

The paper proceeds as follows. Section 2 outlines the traditional GARCH-EVT framework, the BEGE model, and introduces the proposed univariate and bivariate Hawkes point process models. Section 3 describes how VaR forecasts are generated and evaluated. Section 3.1 outlines the equity market and associated IV indices. Section 4 presents in-sample estimation results for the full range of models considered along with the results from tests of forecast accuracy. Section 5 provides concluding comments.

2. Methodology

This section introduces the competing approaches for forecasting extreme losses in the context of VaR predictions. The first is based on the classic GARCH approach where IV is used as an exogenous variable. The specifications considered here are the standard GARCH model of Bollerslev (1986), the GJR-GARCH models of Glosten et al. (1993), and the exponential GARCH (EGARCH) of Nelson (1991). The next approach considered is the BEGE (Bad environment good environment) model of Bekaert et al. (2015) which offers a flexible conditional distribution to describe returns. The approach proposed here utilizes the Hawkes-POT framework introduced in the one-dimensional case by Chavez-Demoulin et al. (2005) which has been employed in a range of empirical applications from modeling equity risk to extreme spikes in electricity prices (Chavez-Demoulin and McGill, 2012; Herrera, 2013; Herrera and González, 2014). Here, the one-dimensional approach is extended to include IV as an exogenous variable. A novel bivariate model is also developed to incorporate the intensity of the occurrence of extreme movements in IV. This approach will uncover potential bi-directional linkages between extreme movements in IV and extreme losses. Results from this analysis will reveal whether using IV itself, or the intensity of its extreme movements, lead to more precise prediction of the intensity and size of extreme equity market losses.

2.1. Conditional mean and volatility models

The conditional mean of the equity market returns is specified as an Auto Regressive Moving Average (ARMA) process

$$r_t = \mu + \sum_{i=1}^{m} a_i r_{t-i} + \sum_{j=1}^{n} b_j \varepsilon_{t-j} + \varepsilon_t. \tag{2.1}$$

Where r_t denotes the return on a stock market index at time t, μ a constant, a_i and b_j describe the autoregressive and moving average coefficients, respectively and ε_t denotes the residual term. The residuals are defined by

$$\varepsilon_t = \eta_t \sqrt{h_t}, \qquad \eta_t \sim iid(0,1), \tag{2.2}$$

where η_t is the standardized residual and h_t is the conditional variance. The GARCH specifications considered for the conditional variances which include IV as an exogenous variable are

$$\text{GARCH}(1,1):\ h_t = \omega + \alpha\varepsilon_{t-1}^2 + \beta h_{t-1} + \gamma IV_{t-1} \tag{2.3}$$

$$\text{GJR}-\text{GARCH}(1,1):\ h_t = \omega + \alpha\varepsilon_{t-1}^2 + \delta \max(0, -\varepsilon_{t-1})^2 + \beta h_{t-1} + \gamma IV_{t-1} \tag{2.4}$$

$$\text{EGARCH}(1,1):\ \ln h_t = \omega + \alpha\varepsilon_{t-1} + \delta(|\varepsilon_{t-1}| - E|\varepsilon_{t-1}|) + \beta \ln h_{t-1} + \gamma \ln IV_{t-1}. \tag{2.5}$$

The GARCH model in Eq. (2.3) corresponds to the standard model of Bollerslev (1986), with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$ and $\gamma \geq 0$ so that the conditional variance $h_t > 0$. The model is stationary if $|\alpha + \beta| < 1$ is ensured. The GJR-GARCH specification in Eq. (2.4) allows the conditional variance to respond asymmetrically to the sign of past returns by means of the parameter δ. Sufficient conditions for $h_t > 0$ are $\omega > 0$, $\alpha + \delta \geq 0$, $\beta \geq 0$ and $\gamma \geq 0$. Finally, the EGARCH specification in Eq. (2.5), allows for asymmetries in volatility if $\delta \neq 0$ while leverage exists if $\delta < 0$ and $\alpha < \delta < -\alpha$. To be consistent with the specification of the conditional variance in Eq. (2.5), IV indices are included in logarithmic form. These three conditional volatility specifications are estimated assuming a Skew Student-t distribution.[1]

The BEGE model of Bekaert et al. (2015) describes the innovations in returns,

$$\begin{aligned} &\varepsilon_t = \sigma_p \omega_{p,t} - \sigma_n \omega_{n,t},\ \text{where} \\ &\omega_{p,t} \sim \widetilde{\Gamma}(p_t, 1),\ \text{and} \\ &\omega_{n,t} \sim \widetilde{\Gamma}(n_t, 1) \end{aligned} \tag{2.6}$$

[1] In a preliminary version of the paper both a conditional Normal, and symmetric student t distribution were also considered. However assuming a skewed student t conditional distribution provides a superior fit to the data. Here the skewness is incorporated into the t-distribution using the method of Fernandez and Steel (1998).