Journal of Banking and Finance 87 (2018) 369–379

Contents lists available at ScienceDirect

Journal of Banking and Finance

journal homepage: www.elsevier.com/locate/jbf

# Detecting money market bubbles

Jan Baldeaux[a], Katja Ignatieva[b,*], Eckhard Platen[c,d]

[a] University of Technology Sydney, Finance Discipline Group, PO Box 123, Broadway, Sydney, NSW-2007, Australia
[b] University of New South Wales Sydney, School of Risk & Actuarial Studies, Business School, Kensington, NSW-2052, Sydney, Australia
[c] University of Technology Sydney, Finance Discipline Group and School of Mathematical and Physical Sciences, PO Box 123, Broadway, Sydney, NSW-2007, Australia
[d] University of Cape Town, Department of Actuarial Science, Faculty of Commerce, Private Bag X3, Rondebosch 7701, Cape Town, South Africa

ARTICLE INFO

*Article history:*
Received 3 April 2017
Accepted 29 October 2017
Available online 2 November 2017

*JEL classification:*
C6
C63
G1
G13

*Keywords:*
Money market bubbles
Strict local martingales
Markov chain Monte Carlo
Stochastic volatility models
Benchmark approach

ABSTRACT

The existence of a self-financing trading strategy that replicates the money market account at a fixed future date at a lower cost than the current value of this account constitutes a money market bubble (MMB). Understanding whether a market exhibits an MMB is crucial, in particular, for derivative pricing. An MMB precludes the existence of a risk-neutral probability measure. The benchmark approach allows to study MMBs and is formulated under the real world probability measure. It does not require the existence of a risk neutral probability measure. Using a range of well-known stochastic volatility models, we study the existence of an MMB in the US economy, and find that the US market exhibits an MMB for all models considered that allow it. This suggests that for derivative pricing and hedging care should be taken when making assumptions pertaining to the existence of a risk-neutral probability measure. Less expensive portfolios are likely to exist for a wide range of long-term derivatives, as typical for pensions.



## 1. Introduction

In this paper, we empirically investigate the potential existence of a money market bubble (MMB) in the US market, as formally introduced in the literature in Heston et al. (2007). An MMB exists if the money market account value at a future time can be replicated at a setup cost lower than the current value of the money market account.[1] In Platen (2002), the potential existence of the type of classical arbitrage generated through an MMB was pointed out. MMBs form part of a growing literature on bubbles, which are studied in both, the economics and the finance literature, see e.g. Cox and Hobson (2005); Gilles (1988); Gilles and LeRoy (1992); Santos and Woodford (1997); Loewenstein and Willard (2000); Jarrow and Madan (2000); Loewenstein and Willard (2006); Heston et al. (2007); Huang and Werner (2000); Hulley (2010); Jarrow et al. (2011); 2007); 2010); Hugonnier (2012) and, in particular, the recent article (Protter, 2013) for an overview of the literature. Heston et al. (2007) defines an asset-price bubble as an asset whose future payoff can be replicated with self-financing portfolio whose present value is lower than the current asset price. As in Heston et al. (2007), we place ourselves in a two asset economy. To be specific, we assume the existence of a well-diversified index and a money market account. As established in Heston et al. (2007), a necessary and sufficient condition for the existence of an MMB is the failure of the existence of an equivalent risk neutral probability measure, also called an equivalent martingale measure (EMM). This means, the Radon-Nikodym derivative of the putative EMM is a strict local martingale and not a true martingale, as assumed under classical no-arbitrage assumptions, see Delbaen and Schachermayer (1994) and Delbaen and Schachermayer (1998). So far though, only (Platen, 2002; Heston et al., 2007) and (Hulley, 2010) have dealt specifically with the phenomenon of money market bubbles. Furthermore, we point out that under Platen's *benchmark approach*, see Platen and Heath (2010), MMBs naturally occur, e.g., under the minimal market model (MMM), see Baldeaux and Platen (2013) and Platen (2002). In case the candidate model rules out MMBs, one can still investigate whether some discounted asset forms a strict local martingale under a respective EMM. If this is the case, then one has detected an asset-price bubble.

* Corresponding author.
*E-mail addresses:* jan.baldeaux@uts.edu.au (J. Baldeaux), k.ignatieva@unsw.edu.au (K. Ignatieva), eckhard.platen@uts.edu.au (E. Platen).

[1] What we investigate in this paper is the *money market account*, which is also sometimes called the savings account, or the risk-free asset, and represents a rollover short-term bond (e.g. T-bill) account.

https://doi.org/10.1016/j.jbankfin.2017.10.017

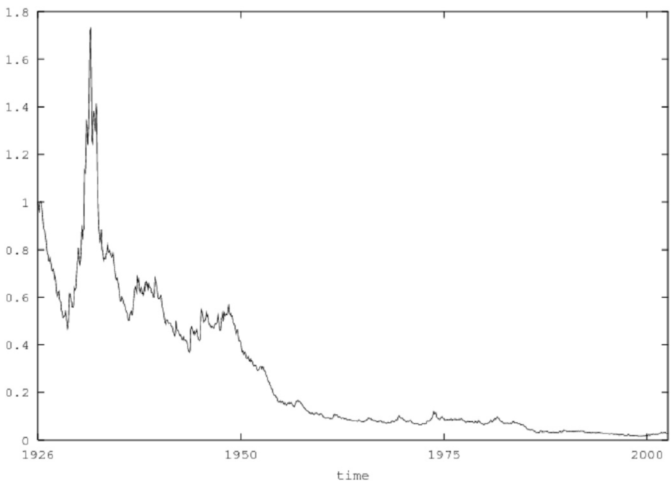


**Fig. 1.** Empirical Radon-Nikodym derivative of the putative EMM for the US economy.

The latter have received much attention in the literature, see e.g. Jarrow et al. (2011); 2007); 2010), and, in particular, the recent article (Protter, 2013). The conditions for the existence of asset-price bubbles are by now well understood: An asset-price bubble exists if and only if the discounted asset value forms a strict local martingale under the EMM. Asset-price bubbles trigger some interesting financial consequences, such as failures of put-call parity or nonconvexity of option prices with respect to the underlying for convex options with convex payoff functions. The current paper examines the fundamental question of the existence of an EMM when aiming for a candidate market model. A motivation for this type of examination is the plot of the US money market account, denominated in units of the total S&P 500 return, shown in Fig. 1, where it is normalised to an initial value of one. For a complete market model that interprets the S&P 500 as numéraire portfolio, as introduced in Long (1990) and will be explained later, Fig. 1 displays an empirical Radon-Nikodym derivative for the putative risk neutral measure for the US market.

The empirical Radon–Nikodym derivative displayed in Fig. 1 is theoretically driftless and, therefore, is a local martingale. It demonstrates a systematic downward trend, indicative of a nonnegative strict local martingale, which is a strict supermartingale, see Platen and Heath (2010), and hence hints at the presence of an MMB. This is opposed to a Radon–Nikodym derivative requested to form a true martingale under classical risk neutral assumption, where the present value is the best forecast of any of its future values. We take Fig. 1 as a motivation for a more thorough investigation of the potential existence of an MMB. By using FX-derivative data, it has been shown in Baldeaux et al. (2015) that an equivalent risk neutral probability measure may not exist for various currency denominations. We aim to employ historical S&P 500 data to provide for the US market alternative evidence in the same direction.

To investigate the existence of an EMM, we require a framework more general than the classical no-arbitrage approach with its risk neutral pricing paradigm. In particular, we require an approach that works under the real world probability measure and is more general than the classical approach. The benchmark approach, see Platen and Heath (2010), offers such a framework and allows us to rigorously examine the potential existence of an EMM. At the heart of the benchmark approach sits the numéraire portfolio (NP), see Long (1990), which when used as a benchmark, makes all nonnegative benchmarked portfolios supermartingales. It is identical to the growth optimal portfolio, which is defined as the portfolio that maximises expected logarithmic utility from terminal wealth, see Kelly (1956). The NP plays an important role in derivative pricing, as it serves as numéraire when pricing contingent claims under the real world probability measure. We alert the reader to the fact that when employing the benchmark approach for derivative pricing, one finds the minimal possible price for contingent claims by employing the *real world pricing formula,PlatenHe10*, which computes the expectation under the real world probability measure using the NP as numéraire.

To summarise our goal, the current paper aims to answer the following main question: *Does a money market bubble most likely exist in the US market?*

We answer this question by connecting two significant streams in the finance literature, namely the literature concerned with the estimation of stochastic volatility models and the literature on the existence of bubbles in the above described sense.

Using Markov Chain Monte Carlo (MCMC) methods, see Christoffersen et al. (2010); Eraker et al. (2003) and references therein, on parametrised stochastic volatility models describing the S&P 500 index, we approach also the second question: *Which is a tractable, parsimonious, yet sufficiently accurate model for the discounted S&P 500?* We adopt the MCMC approach and fit various well-established stochastic volatility models to the S&P 500 index, including the Heston, the 3/2, and the continuous time GARCH model. We refer to Lewis (2000) and Cont (2010) for overviews on stochastic volatility models, including the latter ones. We remark at this point that there are criticisms pertaining to some of these models, as for example discussed in Christoffersen et al. (2009) and Da Fonseca and Grasselli (2011).

We demonstrate that MMBs cannot exist under the Heston model, and this model mainly serves in our study as a popular reference model. For all other stochastic volatility models considered in this paper, we present necessary and sufficient conditions for the existence of an MMB. For these models we find parameter ranges where MMBs exist. Empirically we find that when fitting these models, they strongly hint at the likely existence of MMBs. This suggests that assumptions pertaining to the existence of an EMM should be made with care. Although the main objective of this paper is to investigate empirically MMBs under the selected models, we also compare the model fit based on different selection criteria and find that all models that can generate MMBs lead to superior fitting of the data than the Heston model.

This paper can be seen as building on Heston et al. (2007); Hulley (2010); Hulley and Ruf (2015); Jarrow et al. (2011), and Baldeaux et al. (2014). It is the first paper that tests for the existence of MMBs, which have only been discussed in Platen (2002); Heston et al. (2007); Hulley (2010), in the same way that (Jarrow et al., 2011) was the first paper to investigate empirically the existence of stock price bubbles. We remark that in Baldeaux et al. (2014) local volatility models were fitted to well-diversified indices, and point out that these results support our findings, which suggest the existence of MMBs.

The presence of MMBs violates the ”no free lunch with vanishing risk” (NFLVR) condition, see Delbaen and Schachermayer (1994); 1998), which is the no-arbitrage condition that underpins classical risk neutral pricing. All models we consider are covered under the benchmark approach, which assumes the existence of the NP. The latter condition is equivalent to the ”no unbounded profit with bounded risk” (NUPBR) condition, see Karatzas and Kardaras (2007), which is sufficient for pricing, hedging, risk management and utility maximisation in a general semimartingale framework. It also does not allow creating strictly positive wealth from a nonnegative portfolio with zero initial capital, see Platen and Heath (2010).

From the point of view of classical economic intuition though, a reader acquainted with derivative pricing might challenge the existence of MMBs based on the absence of classical arbitrage: Short-