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Uncertainty on fringe projection technique: A Monte-Carlo-based approach

Jérôme Molimard *, Laurent Navarro

LCG, UMR 5146, École Nationale Supérieure des Mines, CIS-EMSE, CNRS, Saint-Étienne, France

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ABSTRACT

Error estimation on optical full field techniques (OFFT) is millstone in the diffusion of OFFT. The present work describes a generic way to estimate overall error in fringe projection, either due to random sources (phase error, basically related to the quality of the camera and of the fringe extraction algorithm) or the bias (calibration errors). Here, a high level calibration procedure based on pinhole model has been implemented. This model compensates for the divergence effects of both the video-projector and the camera. The work is based on a Monte-Carlo procedure. So far, the complete models of the calibration procedure and of a reference experiment are necessary. Here, the reference experiment consists in multiple step out-of-plane displacement of a plane surface. Main conclusions of this work are: (1) the uncertainties in the calibration procedure lead to a global rotation of the plane, (2) the overall error has been calculated in two situations; the overall error ranges from 104 μm down to 10 μm, (3) the main error source is the phase error even if errors due to the calibration are not always negligible.

1. Introduction

Optical full field techniques (OFFT) are nowadays common tools in university laboratories. Anyway, the confidence on the result obtained is poorly described, and error estimation on OFFT is millstone in their diffusion in industrial world. Usually, the measuring chain is complex, implying optical elements, numerical processing (correlation, phase extraction…) and post-processing (derivation, filtering…). A lot of work has been carried out in order to improve and/or characterize each element of the measuring chain, in particular for image correlation [1,2] or phase extraction [3]. Again, some experimental work gives a global sight on errors, see for example [4,5]. Some work also was done in order to reduce phase errors (see for example [6]). Anyway, overall measurement error still never has been achieved, in particular because of the difficulties to integrate different error sources, among them errors due calibration procedure. Prediction through error model is not straightforward and usually cannot be achieved using standard error propagation rules. Previous works show the efficiency of Monte-Carlo based procedure on specific element of the measuring chain. Description of the error on phase extraction has been provided by Cordero [7]; post-processing derivation has been investigated in the same way [8]. Beside these two general purpose works, a study on 3D ESPI leads to an optimal position of illumination vectors [9]. Anyway, no global prediction approach has been carried out to the best of our knowledge.

Among the different OFFTs, fringe projection is one of the more spread, since its first development [10–12]. Basically, the method renders a shape [5] or a shape variation [13]. Coupled with a 2D correlation system, it can be extended to the measurement of any displacement of a non-flat surface [14–16]. Since it is a non-contacting method, a lot of applications are developed or under development in health engineering (see for example [17–19]).

The present work describes a generic way to estimate overall error in fringe projection, either due to random sources (phase error, basically related to the quality of the camera and of the fringe extraction algorithm) or the bias (calibration errors). Here, a high level calibration procedure based on pinhole model has been implemented [18]. This model compensates for the divergence effects of both the video-projector and the camera. The Monte-Carlo procedure requires complete models of the calibration procedure and of the reference experiment. Here, the reference experiment consists in multiple steps out-of-plane displacement of a plane surface. In order to give boundary values to the overall error, two different situations are investigated: the first one is common macroscopic fringe projection set-up. The second one is a microscopic set-up, optimized for random noise for example considering a larger set of images in the phase extraction.

The paper presents first the Monte-Carlo procedure; then, the specific fringe projection approach is described. Last, the implementation for a given set of experimental conditions is developed, results are analyzed.

* Corresponding author. Tel.: +33 4 77 42 66 48.
E-mail address: molimard@emse.fr (J. Molimard).

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2. Monte-Carlo based uncertainty approach

The uncertainty associated with the result of a measurement is a parameter that characterizes the dispersion of values that can reasonably be attributed to the measurand. Operationally, the dispersion of values of some quantity Q is described by a probability density function (PDF), $f(Q)$. The domain of the PDF consists of all possible values of Q, and its range is in the interval (0,1). If the PDF is known, the estimate of Q is obtained by evaluating the expected value and its standard uncertainty is taken to be equal to the standard deviation [25].

Although obtaining the most appropriate PDF for a particular application is not straightforward, if the measurand Q is related to a set of other quantities $\vec{P}=(P_1\cdots P_{n_p})^T$ through a *measurement model* $Q=M(\vec{P})$, linear or weakly non-linear, the standard uncertainty of Q can be expressed in terms of the standard uncertainties of the *input quantities* $(P_1\cdots P_{n_p})$ by using the so-called law of propagation of uncertainties (LPU) [25,26]. Instead of the LPU, a Monte-Carlo-based technique [22–24] can be applied to linear as well as to nonlinear models, on independent or co-varying error sources.

The Monte-Carlo-based technique requires first assigning probability density functions (PDFs) to each input quantity. Next, a computer algorithm is set up to generate an input vector $\vec{p}_1=(p_1\cdots p_{n_p})^T$; each element p_j of this vector is generated according to the specific PDF assigned to the corresponding quantity P_j. By applying the generated vector $\vec{p}_1$ to the model $Q=M(\vec{P})$, the corresponding output value q_1 can be computed. If the simulating process is repeated N times ($N\gg 1$), the outcome is a series of indications $(q_1\cdots q_N)^t$ whose frequency distribution allows us to identify the PDF of Q, $f(q)$. Then, irrespective of the form of this PDF, the estimate q_e and its associated standard uncertainty $u(q_e)$ can be calculated by

$$q_e=\frac{1}{N}\sum_{l=1}^{N}q_l \tag{1}$$

and

$$u(q_e)=\left(\frac{1}{(N-1)}\sum_{l=1}^{N}(q_l-q_e)^2\right)^{1/2} \tag{2}$$

Knowledge of each element of the $\vec{P}$ vector, in particular the uncertainty level and the PDF shape, directly derives from the experimental knowledge. So far, a good understanding of the whole set-up and procedure is necessary. Here, we suppose that each error source is independent; anyway, cross-dependent inputs are possible.

3. The pin-hole model

The classical pin-hole model characterizes the geometrical relationship between a point in 3D space and its projection on a plane behind another plane in which an aperture was performed. This aperture is supposed to be a point (hence the name pinhole). The Fig. 1 illustrates the principle of the pin-hole model in two dimensions, as the 3D extrapolation is quite simple. O is the aperture and Y is the plane in which the aperture was performed, P is the point in 3D space, x_p and y_p its coordinate. Q is the projection of P in the projection plane Y', f and y_q are its coordinates. Then, the simple equation $y_q=-f\,y_p/x_p$ describes the relationship between a point P in 3D space and its projection Q in 2D plane. The dotted line is called the *projection line*. This model is generally used in shape/displacement measurement systems to account for perspective effects, either for fringe projection [18,27] or stereo-correlation [28]. Note anyway that the following work takes into account perspective effects with an assumption of negligible distortions. In the same way, the optical model does not take into account off-axis arrangement that should be found in many video-projectors. These two points can be considered as the main limitations of the presented work; anyway, the material used in the following is chosen under these hypothesis: dedicated low-distortion lenses, and an in line video-projector.

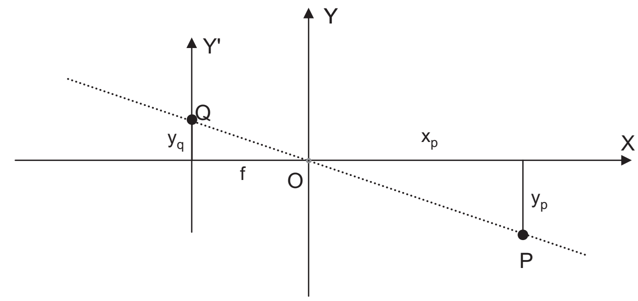

Fig. 1. Illustration of the pin-hole model.

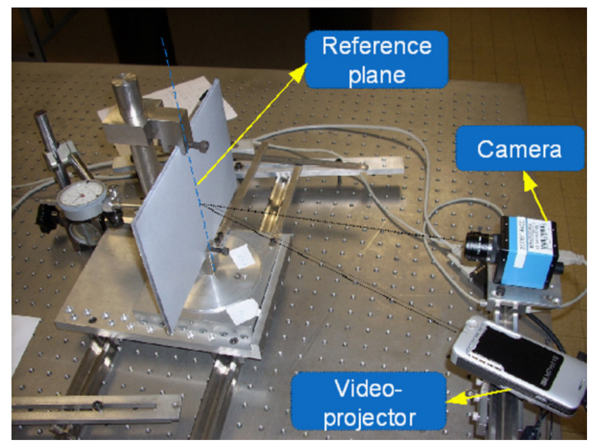

Fig. 2. Optical set-up and calibration test-rig.

4. 3D surface implementation

4.1. Principle of fringe projection

The fringe projection method has already been described by many authors [13,16,17,21]. The physical principle is straightforward: a periodic pattern is projected on an object; the light is diffused by the object and captured by a CCD video-camera. The deformation of the fringes, recorded as phase maps, has a known dependency to the shape of the illuminated object.

Since the fringe projection technique uses the light diffused by an object in order to measure its shape or shape variation, a surface preparation consisting usually in a white paint is sometimes useful. Moreover, in order to observe out-of-plane displacements, the angle between the projected fringes and the observed diffused light must not be null (Fig. 3). Light intensities on an object illuminated by a set of fringes can be described by a periodic function I_{li}, with a perturbation φ corresponding to the object shape:

$$I_{li}(x,y)=I_0(x,y)\left[1+\gamma(x,y)\times\cos\left(\frac{2\pi}{p(x,y)}y+\varphi(x,y)\right)\right] \tag{3}$$

This equation involves an average intensity I_0 and a contrast γ. These values should be constant over the whole map, but some