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# Estimating the reference frame: A smooth twice-differentiable utility function for non-compensatory loss-averse decision-making

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ABSTRACT

Since the introduction of prospect theory, reference-dependence and loss-aversion have become widely acknowledged as important elements affecting decision-making. Nevertheless, establishing and determining reference frames are not extensively analyzed in the literature; rather, in most applications, it is simply assumed that the reference frames can be represented through the *status quo*. This assumption, however, may lead to biased results, as not only the *status quo* affects reference frames, but also previous experiences or expectations, among many others.
Therefore, it would be more appropriate to estimate the reference frame directly as an unobserved latent variable. Unfortunately, current utility functions utilized to depict this kind of behavior are not useful for this purpose, as they are defined piecewise. This work proposes a smooth twice-differentiable utility function that indeed allows estimating reference frames. Further, this function satisfies all major properties of prospect theory. Finally, the approach is tested relying on three case studies. They show that in the context of semi-compensatory loss-averted decision-making reference frames may diverge from the *status quo*.

## 1. Introduction

Since the introduction of prospect theory in 1979 (Kahneman and Tversky, 1979), it has gained numerous supporters, while concepts such as loss-aversion and reference dependence are now common in the behavioral economics literature (Barberis, 2012; Karle et al., 2015). Moreover, it has laid the theoretical foundation for the development of non-compensatory models and theories (e.g. regret theory; Loomes and Sugden, 1982). Challenging the classic economic assumptions about utility and its impact is felt in fields from marketing (Hardie et al., 1993; Ho et al., 2006) to labor economics (Dunn, 1996; Fehr et al., 2009), as well as medicine (Rizzo and Zeckhauser, 2003; Sokol-Hessner et al., 2012), safety (Flügel et al., 2015), and transportation (de Borger and Fosgerau, 2008; Dixit et al., 2015), among many others.

Even though prospect theory was originally developed for addressing choices under risk, it is straightforward to extend its principles to any kind of choice situation involving subjunctive valuations. Basically, prospect theory sustains the existence of reference frames and that the gains and losses relative to these reference points are valued differently by decision makers (Frijns et al., 2008). Thus, in order to correctly assess the behavior, it would be necessary to know the referential frame.

The majority of the literature contributions dealing with loss aversion assumes that the referential points are known *a priori* (e.g. de Borger and Fosgerau, 2008; Thiemann, 2017), which allows for clearly differentiating gains from losses and using different functional

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forms to include them in the expected utility functions. Normally, the reference points are assumed to be equal to the *status quo* (Flügel et al., 2015). At first glance, it appears to be a sound assumption, when it is possible to identify the current conditions (for instance, when dealing with repeated choices). Nevertheless, as several authors correctly point out references do not depend exclusively on the *status quo*, but also on the individuals' previous expectations (Kőszegi and Rabin, 2006), which, in turn, depend on the individuals characteristics and their previous experiences (as it is clearly shown by the value learning literature; Kingsley and Brown, 2010). Moreover, even if it were possible for the modeler to identify for certain these previous expectations (which it is not), the model would still exhibit shortcomings, as empirical evidence indicates that reference frames are also affected by the choice-sets offered to the individuals (Zeelenberg and Pieters, 2007), which is clearly illustrated by the well-known decoy-effect (Huber et al., 1982; Josiam and Hobson, 1995; Guevara and Fukushi, 2016).

A possible way to address the aforementioned issues, assuming loss-averted decision making, would be for the modeler to estimate directly the reference point as a part of the decision model, instead of assuming it *a priori*. This way, the reference would appear as a parameter of the model, representing a change or an inflection point in the utility provided by a given attribute of the decision. Nevertheless, under the usual assumptions for the utility functions - see Maggi (2004) for a good overview on S-shaped utility functions - this approach is not feasible, as they are defined piecewise (with the reference frame representing a discontinuity) and therefore, they are not twice differentiable around zero. Therefore, it is not possible to estimate the position of the reference point, as no gradient (and hessian matrix) exists for it.[1]

This paper introduces an S-shaped utility function that is continuous and twice-differentiable around zero, while still satisfying the main properties of loss-averse utility functions and microeconomic theory. This paper presents an extensive analysis of its properties, such as non-satiation, decreasing marginal utilities, axial asymmetry (which can be calibrated), etc. Such a representation offers multiple possibilities when dealing with loss-averse decision-making as it not only allows estimating the reference frames, but also how frames are affected by the individuals' characteristics and/or the offered choice-sets. The function exhibits a simple structure, which allows for an easy implementation in discrete choice models. Finally, the proposed approach is tested with the help of three case studies that indicate that reference frames may indeed diverge from the *status quo*.

## 2. Theoretical framework

Decision-making is usually approached from a utilitarian perspective. It suggests that decision-makers $q$ will opt for the alternative $i$, belonging to a given choice-set $A_q$, that maximize their expected utility $U_{iq}$. Random Utility Theory (Thurstone, 1927; McFadden, 1974) postulates that this utility can be represented as the sum of a representative component and an error term ($\varepsilon$). If we assume additive linearity, it leads to the following expression:

$$U = X \cdot \beta + \varepsilon, \tag{1}$$

where $X$ is a multidimensional matrix, whose dimensions represent individuals, alternatives and the observed attributes and characteristics of the aforementioned alternatives and individuals, respectively. $\beta$ is a matrix of parameters to be estimated (whose rows are associated with the different elements of $X$, while the columns represent the different alternatives in the choice-set) and $U$ and $\varepsilon$ are matrixes, whose rows and columns represent individuals and alternatives, respectively.[2] The error $\varepsilon$ can follow any desired distribution, but it is customary assumed to be i.i.d. Extreme-Value Type 1 (EV1) distributed, which leads to the well-known Multinomial Logit model (Domencich and McFadden, 1975; MNL). Finally, the choices by the decision-makers $q$ are represented by the matrix $Y$ (same dimensions as $U$), whose elements take the value of one if the alternative is selected, and zero otherwise.

Obviously eq. (1) assumes a linear impact of the explanatory variables $X$ over the utility function. This restriction, however, can be easily lifted by assuming that the elements of $X$ represent, in fact, any possible transformation of the observed variables (e.g. an exponential or a Box-Cox transformation; Ortúzar and Willumsen, 2011). Hence, eq. (1) can be expressed in the following fashion.

$$U = f(X) \cdot \beta + \varepsilon, \tag{2}$$

where $f(X)$ is a matrix function of $X$. This representation is quite convenient to characterize real behavior as economic theory suggests that the marginal utilities of a given good are decreasing. Thus:

$$\frac{\partial^2 U}{\partial x^2} < 0, \tag{3}$$

where x is a given element of X (attribute of the alternative) that can be considered to be a good in accordance to the Lancaster's principles (Lancaster, 1966). Therefore, it would be adequate to consider monotonically non-decreasing and concave functions for $f(X)$ (obviously the function should have a negative sign if the considered attribute is an economic bad, such as the price).

Nevertheless, per definition, a discrete choice implies necessarily a trade-off. Thus, every time an individual makes a decision, they

[1] Nevertheless Kőszegi and Rabin (2006, 2007) show that it is still possible to derive a functional model, although it still exhibits discontinuities and a degree of complexity that makes it impractical for most discrete choice modeling applications.

[2] The multidimensional matrix formulation differs from the standard notation considering individuals and alternatives as sub-indexes (Ben-Akiva and Lerman, 1985; Train, 2009). Both specifications are, however, absolutely equivalent.