Journal of Choice Modelling xxx (2017) 1–12

Contents lists available at ScienceDirect

Journal of Choice Modelling

journal homepage: www.elsevier.com/locate/jocm

Quantum paradigm of probability amplitude and complex utility in entangled discrete choice modeling

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A R T I C L E   I N F O

Keywords:
DCM
Quantum probability amplitude
MNL
Complex utility
Entangled choices
BWS probability structure

A B S T R A C T

The main idea of this paper is motivated by a paradigm from quantum physics where the probability amplitude is built as a superposition of the wave functions of states, and the squared modulus of amplitude defines the probability of state membership. Similar linear aggregates are used in classical physics for description of wave interference effects. In contrast to regular techniques of probability estimation in social-economic research (such as logistic regression, multinomial-logit (MNL), discrete choice modeling (DCM), conjoint, best-worst scaling (BWS), and other methods), the proposed approach of probability amplitude modeling permits finding choice probabilities themselves and demonstrates possible interference phenomena of entanglement of different choices. Particularly, a BWS example evaluated by complex utility MNL demonstrates how a choice of each item is composed from its pure-state and mix-state probabilities. The obtained numerical results are supportive of theoretical considerations and practical applications of the probability amplitude modeling, and can serve for better understanding and evaluation of choice decisions.

1. Introduction

This work is motivated by an attempt to implement a quantum-like paradigm in some applied research projects dealing with constructing the probability amplitude and complex utilities for discrete choice modeling (DCM). The DCM is probably the most popular tool in applied social and economic sciences used for estimation of utility parameters and choice preferences among multiple alternatives. DCM is usually performed via the multinomial-logit (MNL) modeling originated in McFadden (1973) and developed in numerous subsequent works (McFadden and Richter, 1990; Louviere et al., 2000; Train, 2003; Lipovetsky, 2011, 2014, 2015; Dellaert et al., 2017). Other important developments based on DCM include conjoint modeling and best-worst scaling (BWS) proposed by Louviere for evaluating probability of choice among many compared items (Anderson et al., 1992; Orme, 2010; Lipovetsky and Conklin, 2014a,b; 2018; Louviere et al., 2015; Marley et al., 2016).

Quantum mechanics (QM) presents a vast area of ideas, methods, and techniques established over a hundred years, and in the last years QM tools have been tried in other fields of human activity and interests, including social and political studies, economics and financial markets, psychology and decision making. Developed at first for quantum description of information and communications, computers and artificial intelligence, causality modeling and Bayesian networks (Jaynes, 1957; Tucci, 1995; Schneidman et al., 2006; Lakshminarayan et al., 2007; Leifer and Poulin, 2008; Ying, 2010; Moreira and Wichert, 2016; Brandenburger and La Mura, 2016; Evans, 2017), these ideas of information processing have been extended to investigation of human probability judgements and cognition, and to social science problems (e.g., Busemeyer and Bruza, 2012; Khrennikov and Haven, 2013; Tegmark, 2015; Wang and

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https://doi.org/10.1016/j.jocm.2017.10.003
Received 7 July 2017; Received in revised form 24 October 2017; Accepted 25 October 2017
Available online xxxx

Busemeyer, 2016; Aroyo and Welty, 2017; Furioli et al., 2017). A noticeable development had been reached in works by Camparo (2013), and Camparo and Camparo (2013) for application of the Likert scales with the mixed distribution of the levels. Various ideas and techniques from physics were applied to socio-economics studies in numerous works creating related fields of sociophysics, socio-dynamics, econophysics, and mediaphysics (see, for example, Weidlich, 2000; McCauley, 2004; Chatterjee et al., 2005; Kuznetsov and Mandel, 2007; Lipovetsky, 2010; Galam, 2012).

The current paper is based on some ideas and techniques of QM applied in the so-called quantum-like paradigm. Let us start with a brief description of QM concepts and their applications to social, economics, and psychological studies collected in a recent handbook edited by Haven and Khrennikov (2017) (further called HK). HK contains sixteen papers written by leading experts who give a panoramic view of modern QM developments for application to the problems of choices made by decision makers in social and economic fields, with bibliography covering in total hundreds of references. The "applications of quantum formalism outside of physics is known as the *quantum-like paradigm*" (HK, p. v), and it does not require knowledge of the actual physical processes (for example, in applications to cognitive and behavioral processes). Such approach rather corresponds to modeling outcomes of possible observations and measurements of compound phenomena where "a deep uncertainty is represented by using two basic structures of the mathematical formalism of quantum theory, *superposition* and *entanglement* of states" (HK, p. vi). Due to Max Born interpretation of QM, the squared module of complex wave function of the states' superposition yields the probability of objects to belong to different states, which opens a possibility to apply QM tools for investigation of other complex objects, particularly, in social and economic studies. Quantum-like observables and states correspond to questions/tasks and belief states of respondents or social groups, and QM formalism is used as a working tool to calculate probabilities, leaving aside interpretation for each particular application (HK, Ch.1). Quantization in financial and behavioral economics permits QM formulation of Fisher information and opens some useful interpretations with application to risk and volatility which come together with uncertainty (HK, Ch.2). Human decisions often demonstrate violation of probability theory and Boolean logic, and QM could suggest solutions of a mixed strategy with probabilities found as squared modules of aggregates of pure states (HK, Ch.3). Interference of a person's states of mind can be expressed similarly to interference of waves in the classical optics, and "the so-called '*quantum superposition*' is nothing but the expression of the fact that the player has not 'made up his or her mind' until he or she is asked to do so in an experiment" (HK, Ch.4, p.71). Following the views of QM founder Niels Bohr, it is possible to indicate that "everything that can be said about nature is obtained from measurements … the cognitive and social sciences can also be treated as theories of measurements" (HK, Ch.5, pp.75–76), so the quantum instruments can be applied to the direct and indirect measurements in the cognitive and social sciences. Human perception depends on the experimental settings and could demonstrate violation of classical probability laws of additivity, so it rather corresponds to quantum formalization of states and their interference yielding the probability distributions which can be compared with experimental facts (HK, Ch.6). Concerning choices, M. Born's concept of quantum probability reduces to Kolmogorov's set-theoretic probability only in special cases, and a non-classical multiple discrete choice probability can oscillate in time and by other characteristics because it reveals the probability as in spontaneous decisions of the respondents based on their previous sets of views and attitudes to different issues, so "irrelevant events can have a small but detectable effect on voting behavior" and a subject can "be one thing and another at the same time" (HK, Ch.7, p.131, and p.135). Not only the core law of total probability in Kolmogorov's set theory but also the Bayesian update of respondents' preferences can be violated, so the uncertain outcomes require a modeling through superposition and entanglement as in QM where choice probability is defined with additional items of interference between the states and the interference magnitude parameters are calculated from the data (HK, Ch.8). Concerning human perception and its adequate description, some recent work showed that human mind operates with concepts not by rules of classical logic even when those are simple conjunctions or disjunctions but rather by a context situation, so in cognitive modeling the superposition of concepts reveals a state of mind which includes interference terms of entanglement in concept formation (HK, Ch.9). Prospect theory (Kahneman and Tversky, 1972, 1973; Kahneman et al., 1982; Tversky and Kahneman, 1974, 1983; Gilovich et al., 2002) explains some observations in human decisions better than classical probability theory. For instance, the Savage's rational decision-making concept called the sure-thing principle does not always hold but depends on contextual effects (HK, Ch.10; see also Molin and Timmermans, 2010; Dzhafarov et al., 2015). Preferences in prospect theory can be described by a decision theory model, based on mixing an agent's states of mind into their superposition, where the "model induces a game among potential incarnations of the individuals … these … represent conflicting desires or propensities to act" (HK, Ch.11, p.242). "A large number of empirical studies have shown that order of information plays a critical role in human judgements" (HK, Ch.12, p.259), and modeling of processes by which people reason about causes and effects could be performed by quantum generalization of Bayesian causal networks with classical probability substituted by calculations of probability via QM amplitudes. Therefore, the constructive cognitive processes in judgement and decision making can be adequately modeled by quantum probability theory which leads to such intriguing questions as "Does consciousness have causal powers? … Are our actions at least sometimes determined by our conscious free will?" (HK, Ch.14, pp. 293–294). Thus, the QM methods are tried in "multidisciplinary research unifying quantum information and probability, open quantum systems and the foundation of probability with … cognitive psychology, decision-making, economics and finance, and social science and politics" (HK, Ch.15, p.321). Such QM principles as discreteness, probabilistic or statistical nature of processes, Bohr's principle of correspondence of quantum to classical laws, and principles of information processing can be discussed in their relation to the non-physical grounds in psychology, socio-economics, and other fields (HK, Ch.16). Particularly, Bohr's semi-classical predictions can be considered for big quantum numbers and transition to continuum, where, for instance, superposition of discrete and continuous states can yield resonance processes of Fano kind (Lipovetsky and Senashenko, 1972, 1973, 1974; Reyes et al., 2017).

Although these and various other theoretical works on possible QM applications in socio-economic and psychological sciences have been considered, much less is known on real performance of quantum ideas and techniques in real-world projects based on available