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A new approach to calculating welfare measures in Kuhn-Tucker demand models

Patrick Lloyd-Smith

Department of Agricultural and Resource Economics, University of Saskatchewan, 51 Campus Dr, Saskatoon, SK S7M 5A8, Canada

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ABSTRACT

I develop a new approach to calculating welfare measures in Kuhn-Tucker consumer demand models that uses the analytical properties of the Multiple Discrete-Continuous Extreme Value (MDCEV) specification. I adapt Pinjari and Bhat's (2011) Marshallian demand forecasting routine to calculate Hicksian demands that are useful for computing welfare measures. Simulations demonstrate that this new approach substantially reduces computational time relative to the existing approach using a numerical bisection routine. The new approach performs best relative to the numerical bisection routine if i) a γ-profile utility function is specified, ii) the number of choice alternatives available is high, or iii) the average number of chosen alternatives is low.

1. Introduction

Many individual choice contexts can be characterized by both extensive (i.e. what alternative to choose) as well as intensive (i.e. how much of an alternative to consume) margins where individuals are not restricted to only choosing a single alternative. These multiple discrete-continuous (MDC) choice situations are ubiquitous, arising in transportation, marketing, and decisions regarding environmental resources.[1] Kuhn-Tucker (KT) consumer demand models are often employed to analyze these MDC situations and substantial progress has been made on improving the econometric modeling structures. One reason cited for the lack of widespread use of these KT models is that applying these models for welfare analysis is not straightforward (von Haefen and Phaneuf, 2005; Bhat and Pinjari, 2014). This issue is especially relevant in applying these models to studying decisions regarding environmental resources where producing welfare estimates is often the main purpose of the research.

Computing exact welfare measures from individual demand models face many difficulties.[2] While the theoretically correct welfare measures are based on Hicksian demands, which hold utility constant and can be used to compute compensating and equivalent variation, analysts often use demand models that provide estimates of Marshallian demands and their associated consumer surplus welfare measure (Bockstael and McConnell, 2007; Laird, 2010). A large theoretical and empirical literature has focused on the appropriateness of calculating Hicksian welfare measures from Marshallian demands which has motivated the development of a diverse set of approaches that use various approximations and assumptions on the structure of equations (Willig, 1976; Jara-Diaz and Videla, 1990; Bockstael and McConnell, 2007; Dalya et al., 2008; Laird, 2010). Overall the literature provides mixed evidence on whether consumer surplus is a good proxy for Hicksian welfare measures. Calculating Hicksian demands directly avoids the approximations and assumptions that are required when starting with Marshallian demands. One of the advantages of the KT modeling framework is that the utility function is explicitly specified, which allows for the direct computation of exact Hicksian welfare measures. This fact motivates

E-mail address: plloydsmith@gmail.com.

[1] Bhat and Pinjari (2014) review the MDC choice model literature and discuss relevant empirical applications.

[2] For a textbook treatment of these issues, see Bockstael and McConnell (2007).

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the approach developed in this paper.

This paper describes a new approach to calculating Hicksian welfare measures in KT consumer demand models. The main difficulty in calculating the optimal consumption quantities for individuals in KT models is that once the model parameters are estimated, a constrained, non-linear optimization problem needs to be solved. The existing iterative approach, using a numerical bisection routine (von Haefen, 2007), works well in most applications and is computationally more efficient than earlier enumerative approaches, where every possible solution is checked (Phaneuf et al., 2000). However, its iterative nature is undesirable in more data intensive applications and relies on the arbitrary choice of a stopping criteria. The new approach presented here uses analytical properties and expressions of the Multiple Discrete-Continuous Extreme Value (MDCEV) utility specification (Bhat, 2008) to significantly reduce computation time. I adapt Pinjari and Bhat's (2011) Marshallian demand forecasting routine to calculate Hicksian demands that can be used to compute exact welfare measures. Simulations using a real data set suggest that using the new algorithms can reduce computation time 3- to 12-fold compared to the existing iterative approach. Experiments conducted using simulated data also demonstrate that the new approach's relative computational performance is best when the number of choice alternatives available is high or the average number of chosen alternatives is low.

2. The individual's expenditure minimization problem

I start by considering the general MDCEV utility function as in Bhat (2008):

$$U(x) = \sum_{k=2}^{K} \frac{\gamma_k}{\alpha_k}\psi_k\left[\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right] + \frac{\psi_1}{\alpha_1}x_1^{\alpha_1}. \tag{1}$$

where x_k is the amount of K alternatives available to the decision maker and x_1 is the numeraire or "outside" good that is always consumed in positive quantities. To be consistent with the properties of a utility function $\gamma_k > 0$, $\psi_k > 0$ and α_k, $\alpha_1 \leq 1$ for all k are required for this function (Bhat, 2008). Although the standard assumption is to assume the price of the numeraire is equal to one, I use p_1 throughout this paper for clarity. The ψ_k, γ_k, and α's terms are structural parameters of the utility function and Bhat (2008) provides a thorough overview of the interpretation of these parameters. In brief, ψ_k is the marginal utility of alternative k when $x_k = 0$, the α's are satiation parameters and control the rate of diminishing marginal utility of additional consumption of an alternative, and γ_k shifts the underlying indifference curves which allows for corner solutions (i.e. zero consumption levels for certain alternatives).

Individuals are assumed to maximize utility given by Equation (1) subject to a linear budget constraint and non-negativity constraints on x_k. Pinjari and Bhat (2011) solve this consumer problem to yield analytical expressions for Marshallian demands. However, for welfare analysis, we are interested in Hicksian demands and thus I set up the consumer's expenditure minimization problem holding utility constant at the baseline level ($\overline{U}$). Specifically, the consumer's expenditure minimization problem is

$$\min_{\sum_{k=1}^{K} x_k} \; E = \sum_{k=1}^{K} p_k x_k \; \text{ subject to } \; U(x) = \overline{U}$$

and non-negativity constraints on the Hicksian demand consumption, x_k. The Lagrangian equation is then given by

$$L = \sum_{k=1}^{K} p_k x_k + \lambda^E\left[\overline{U} - \sum_{k=2}^{K} \frac{\gamma_k}{\alpha_k}\psi_k\left[\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right] - \frac{\psi_1}{\alpha_1}(x_1)^{\alpha_1}\right],$$

where λ^E is the Lagrangian multiplier associated with the baseline utility constraint. The resulting KT first-order conditions for optimal expenditures are given by:

$$\frac{\partial L}{\partial x_1} = p_1 - \lambda^E\psi_1(x_1)^{\alpha_1 - 1} = 0 \tag{2}$$

$$\frac{\partial L}{\partial x_k} = p_k - \lambda^E\psi_k\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k - 1} = 0, \; \text{ if } \; x_k > 0, \; k = 2, \ldots K, \tag{3}$$

$$\frac{\partial L}{\partial x_k} = p_k - \lambda^E\psi_k\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k - 1} > 0, \; \text{ if } \; x_k = 0, \; k = 2, \ldots K, \tag{4}$$

$$\frac{\partial L}{\partial \lambda^E} = \overline{U} - \sum_{k=2}^{K} \frac{\gamma_k}{\alpha_k}\psi_k\left[\left(\frac{x_k}{\gamma_k}+1\right)^{\alpha_k} - 1\right] - \frac{\psi_1}{\alpha_1}x_1^{\alpha_1} = 0. \tag{5}$$

These first-order conditions can be used to derive Hicksian demands and welfare measures. The Hicksian compensating surplus (CS^H) for a change in price and quality from baseline levels p^0 and q^0 to new 'policy' levels p^1 and q^1 is defined implicitly using an indirect utility function