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Theoretical analysis of sensitivity-tunable total-internal-reflection heterodyne interferometer

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ABSTRACT

This study presents a theoretical analysis of a sensitivity-tunable total-internal-reflection (TIR) interferometer. The interferometer consists of a heterodyne light source, an isosceles right-angle prism, and some polarization components. When a half-wave plate and two quarter-wave plates with proper azimuth angles are arranged in the tested arm, the final phase difference of the interference signal is associated with the azimuth angle of the transmission axis of the analyzer in the arm. Numerical calculations demonstrated that phase sensitivity and measuring range are controllable by tuning the azimuth angle of the analyzer. The feasibility of the measuring method was demonstrated by the experiment results. Our method of measurement has implicational merits of both common-path interferometry and heterodyne interferometry.

1. Introduction

The optical phenomenon of total-internal reflection (TIR) occurs when a ray of light strikes an interface between a denser and a rarer medium at an angle larger than the critical angle with regard to the normal of the surface. This phenomenon causes the reflected wave to experience the phase shifts, which relates to the angle of incidence and the refractive indices (RI) of the media. Several optical sensing technologies have used the characteristics in detecting the changes of physical parameters, such as refractive index change in solutions or gases [1–3], two-dimensional refractive index distribution of optical materials [4], surface and thin-film analysis [5,6], chromatic dispersion in optical materials [7], angular variation in machine tools [8–11], and displacement in micro-mechanical electronic systems [12]. These methods produced excellent measurement results. Although a number of these methods can measure slight variations of the parameters and achieve high detection sensitivity using a multiple total-internal-reflection apparatus, the apparatus has the disadvantages of a large volume and weight [1,8–12]. These factors cause difficulty in using the TIR apparatus in some space-limited areas. In addition, to avoid the 2π phase jumping, the systems provide only a narrow measurement range, which makes them unsuitable for tracing large parameter changes. To detect such changes, the apparatus must be replaced by an apparatus with lower numbers of reflections to expand the measurable range [2–7]. This process causes the setup to be rearranged and recalibrated, and adds the operational complexity. To improve these disadvantages, a method that can be used in various measuring conditions must be developed. However, to our knowledge, no references were reported for this purpose. Consequently, we designed a sensitivity-tunable heterodyne interferometer using total-internal-reflection effect and derived some theoretical equations. A linearly heterodyne light beam is guided to propagate through a half-wave plate and two quarter-wave plates, and, subsequently, enters a TIR apparatus. The completely reflected light finally passes an analyzer to extract the interference signal of *p*- and *s*- polarized light. When the azimuth angles of these wave plates are chosen appropriately, the final phase difference of the interference signal is associated with the azimuth angle of the transmission axis of the analyzer, yielding the sensitivity-tunable functionality. Theoretical simulations demonstrated that the angular sensitivity is tunable from 0 to at least several dozen times the sensitivity obtained with single total internal reflection near the critical angle, and RI sensitivity ranges from 0°/RIU to $1.2\times10^{5\circ}$/RIU. Additionally, the analytic results also indicated that the smallest RI error 8.2×10^{-6} is obtainable under the highest RI sensitivity condition, and the error will increase when the RI sensitivity decreases. The experimental results of phase differences obtained by this technique were effectively confirmed by the theoretical analysis. In addition to the tunable phase sensitivity and small size TIR apparatus, this method also has the advantages of simple structure, and high stability and resolution because of its common-path configuration and heterodyne phase measurement.

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2. Principle

Fig. 1 displays the schematic diagram of a sensitivity-tunable TIR heterodyne interferometer. For convenience, the $+z$ axis is set in the direction of propagation of light and the x axis is perpendicular to the plane of the paper. After the laser light passes through a half-wave plate H_1, the polarization plane of the light is at an angle θ_p from the x axis and has the Jones vector

$$E_i = \begin{pmatrix} \cos\theta_p \\ \sin\theta_p \end{pmatrix}. \tag{1}$$

The Jones vector, after emergence from an electro-optic modulator (EO) driven at an angular frequency ω, is

$$E_{i1} = \begin{pmatrix} e^{i(\omega t/2)} & 0 \\ 0 & e^{-i(\omega t/2)} \end{pmatrix}\begin{pmatrix} \cos\theta_p \\ \sin\theta_p \end{pmatrix} = \begin{pmatrix} e^{i(\omega t/2)}\cos\theta_p \\ e^{-i(\omega t/2)}\sin\theta_p \end{pmatrix}. \tag{2}$$

The light is incident on a beam splitter (BS) and divided into two parts, a reflected and a transmitted beam. The reflected beam proceeds through an analyzer AN$_r$, whose transmission axis is at α to the x-axis and produces the amplitude E_r, which ends at a photo detector D_r:

$$E_r = \begin{pmatrix} \cos^2\alpha & \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha & \sin^2\alpha \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi_{BS}} \end{pmatrix}\begin{pmatrix} e^{i(\omega t/2)}\cos\theta_p \\ e^{-i(\omega t/2)}\sin\theta_p \end{pmatrix}$$
$$= \left[e^{i(\omega t/2)}\cos\alpha\cos\theta_p + e^{-i((\omega t/2)-\phi_{BS})}\sin\alpha\sin\theta_p\right]\begin{pmatrix} \cos\alpha \\ \sin\alpha \end{pmatrix}, \tag{3}$$

and the intensity detected by D_r is

$$I_r = \tfrac{1}{2}\left[2(\cos\alpha\cos\theta_p)^2 + 2(\sin\alpha\sin\theta_p)^2 + \sin(2\theta_p)\sin(2\alpha)\cos(\omega t - \phi_{BS})\right]. \tag{4}$$

where I_r is the reference signal and ϕ_{BS} the phase difference between p- and s- polarizations produced by the reflection at BS. Conversely, the transmitted beam passes through a half-wave plate H_2 (the fast axis at $\Delta/2$ to the x-axis), two quarter-wave plates Q_1 and Q_2 (the slow axes at 45° and 0° with respect to x-axis, respectively), and is subsequently incident at θ_i on one side of an isosceles right-angle prism P (located on a rotational stage) with a refractive index of n_p. The light beam penetrates the prism at an angle of incidence θ_1 onto the interface between the prism and a medium with a refractive index of n. When θ_i exceeds θ_{ic}, which is the angle that makes θ_1 equal to the critical angle θ_{1c}, the light is totally reflected at the interface, and the reflected light beam travels through a analyzer AN$_t$ (with the transmission axis being at β to the x-axis) for interference. The amplitude E_{i1} becomes E_t and is detected by D_t:

$$E_t = \begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix}\begin{pmatrix} t_a e^{-i(\delta_t/2)} & 0 \\ 0 & t_b e^{i(\delta_t/2)} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}\frac{1}{\sqrt{2}}$$
$$\times\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}\begin{pmatrix} \cos\Delta & \sin\Delta \\ \sin\Delta & -\cos\Delta \end{pmatrix}\begin{pmatrix} e^{i(\omega t/2)}\cos\theta_p \\ e^{-i(\omega t/2)}\sin\theta_p \end{pmatrix}$$
$$= \left(e^{i((\omega t/2)+\phi_1)}A_{t1}\cos\theta_p + e^{-i((\omega t/2)+\phi_1-\phi_2-(\pi/2))}A_{t2}\sin\theta_p\right)\begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}, \tag{5}$$

Therefore, the intensity of this beam is

$$I_t = |E_t|^2 = [(A_{t1}\cos\theta_p)^2 + (A_{t2}\sin\theta_p)^2 + A_{t1}A_{t2}\sin 2\theta_p \times \cos(\omega t + \phi - \pi/2)], \tag{6}$$

with

$$A_{t1} = \sqrt{\tfrac{1}{2}\left[(t_a\cos\beta)^2 + (t_b\sin\beta)^2 + t_a t_b \sin 2\beta \times \cos(2\Delta + \delta_t)\right]}, \tag{7}$$

$$A_{t2} = \sqrt{\tfrac{1}{2}\left[(t_a\cos\beta)^2 + (t_b\sin\beta)^2 - t_a t_b \sin 2\beta \times \cos(2\Delta + \delta_t)\right]}, \tag{8}$$

$$\phi = \phi_1 - \phi_2, \tag{9}$$

$$\phi_1 = \tan^{-1}[-\tan(45^\circ - \sigma) \times \tan(\Delta + \delta_t/2)], \tag{10}$$

$$\phi_2 = \tan^{-1}[-\tan(45^\circ + \sigma) \times \tan(\Delta + \delta_t/2)], \tag{11}$$

$$\delta_t = 2\tan^{-1}\left\{\frac{\sqrt{\sin^2[45^\circ + \sin^{-1}(\sin\theta_i/n_p))] - (n/n_p)^2}}{\tan[45^\circ + \sin^{-1}(\sin\theta_i/n_p)] \times \sin[45^\circ + \sin^{-1}(\sin\theta_i/n_p)]}\right\}, \tag{12}$$

$$\tan\sigma = \frac{t_b}{t_a}\tan\beta, \tag{13}$$

$$t_a = t_p t_p' \tag{14}$$

$$t_b = t_s t_s' \tag{15}$$

and

$$t_p = \frac{2\cos\theta_i}{(\cos\theta_i/n_p) + \sqrt{1 - (\cos\theta_i/n_p)^2}}, \tag{16}$$

$$t_s = \frac{2\cos\theta_i}{\cos\theta_i + n\sqrt{1 - (\cos\theta_i/n_p)^2}}, \tag{17}$$

$$t_p' = \frac{(2/n_p) \times \sqrt{1 - (\cos\theta_i/n_p)^2}}{(\cos\theta_i/n_p) + \sqrt{1 - (\cos\theta_i/n_p)^2}}, \tag{18}$$

$$t_s' = \frac{(2/n_p) \times \sqrt{1 - (\cos\theta_i/n_p)^2}}{\cos\theta_i + (1/n_p) \times \sqrt{1 - (\cos\theta_i/n_p)^2}}, \tag{19}$$

where the values of (t_p, t_s) and (t_p', t_s') are the transmission coefficients at the air-prism and the prism-air interface, respectively, and δ_t is the phase difference between s- and p- polarizations of one total-internal reflection at the prism-medium interface [12–14]. The constant Δ in Eqs. (10) and (11) is induced by using the wave plates H_2, Q_1, and Q_2 in the system, and it is changeable by rotating the azimuth angle of the fast axis of H_2. In our method, the constant Δ is used to vary the phase level of $\delta_t/2$, and must be

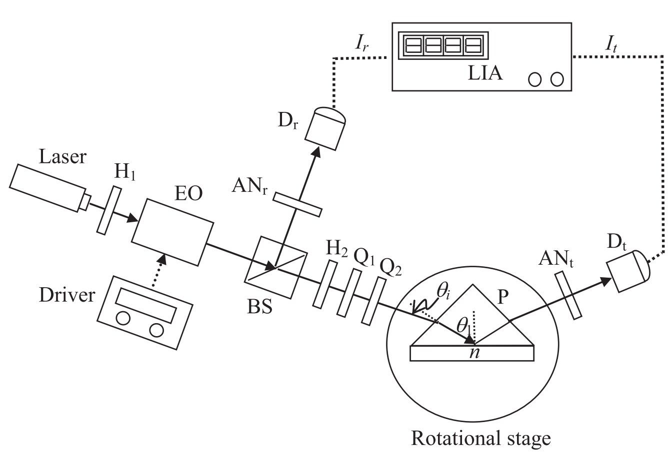

Fig. 1. Schematic diagram of the sensitivity-tunable heterodyne interferometer. EO: electro-optic modulator, BS: beam splitter, H: half-wave plate, Q: quarter-wave plate, P: Prism, AN: analyzer, D: photo detector, LIA: lock-in amplifier.