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Nonparametric identification of the distribution of random coefficients in binary response static games of complete information☆

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ABSTRACT

This paper studies binary response static games of complete information allowing complex heterogeneity through a random coefficients specification. The main result of the paper establishes nonparametric point identification of the joint density of all random coefficients except those on interaction effects. Under additional independence assumptions, we identify the joint density of the interaction coefficients. Moreover, we prove that in the presence of covariates that are common to both players, the player-specific coefficient densities are identified, while the joint density of all random coefficients is not point-identified. However, we do provide bounds on counterfactual probabilities that involve this joint density.

1. Introduction

Motivation. Unobserved heterogeneity across cross sectional units is ubiquitous in situations of strategic interaction. The decisions of airlines to enter a given local market, for instance, may be dramatically influenced by unobservable factors not captured by observables like market size or average income. Similarly, there may be profound differences in the work and retirement decisions of married couples that are not sufficiently explained by observed variables like age, number of children, or religious background. Yet, understanding the extent of these differences across many local markets or many couples is crucially important for making effective policy decisions.

In this paper, we adopt a random coefficients approach to model such heterogeneity of players across different cross sectional units, e.g. local markets or couples. We consider the most basic model of strategic interaction in a binary response,

☆ This paper contains parts of the identification section of Dunker, Hoderlein, and Kaido (2014) "Random Coefficients in Static Games of Complete Information," (DHK, henceforth). The estimation part of DHK will be part of a companion paper.

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two player one-shot complete (perfect) information game. To this end, we study a binary response linear index dummy endogenous variable simultaneous equation model. This model has been extensively analyzed with nonrandom coefficients and a scalar unobservable, see Amemiya (1974), Heckman (1978), Bjorn and Vuong (1985), Bresnahan and Reiss (1990, 1991), Berry (1992) and Tamer (2003). More recently, this line of work has been extended by Kline (2015) to allow for a scalar heterogeneous random parameter on the interaction term. Fox and Lazzati (2017) consider a complete information game with multiple players and study its relation to the demand of bundles, while allowing for unobservable heterogeneity as in Kline (2015). In contrast to all these references, we focus on the two player game with possibly high dimensional unobservable heterogeneity.

To formalize our approach, under suitable assumptions on the solution of the game our structural model maps into the following reduced form system of equations:

$$
\begin{aligned}
Y_1 &= \mathbf{1}\{(B_1 + \Delta_1 Y_2)X_1 - Z_1 < 0\} \\
Y_2 &= \mathbf{1}\{(B_2 + \Delta_2 Y_1)X_2 - Z_2 < 0\}.
\end{aligned}
\tag{1.1}
$$

Here Y_j denotes a binary action that player j may take. In the above retirement example, $Y_1 = 1$ denotes the decision of spouse 1 to retire. In the market entry example, $Y_2 = 0$ denotes the decision of firm 2 not to enter the market. We assume that this decision, for each player $j = 1, 2$, is determined by whether the latent utility $Y_j^* = Z_j - (B_j + \Delta_j Y_{-j})X_j$ is above or below a threshold normalized to be zero; if the utility is above the zero threshold, (equivalently, if $(B_j + \Delta_j Y_{-j})X_j - Z_j < 0$), player j chooses $Y_j = 1$. This utility is partially determined by observables, specifically, the covariates $\tilde{X}_j = (X_j, Z_j)$, where X_j includes a constant and Z_j is a scalar,[1] as well as the action of the other player, Y_{-j}. However, it is also partially determined by the unobservables B_j and Δ_j, which determine how observable covariates, as well as the action of the other player influence the latent utility.

The key innovation in this paper is allowing all the variables, including the unobserved parameters, to vary across the population, thus adopting a perspective of extensive heterogeneity. We then provide a framework in which we establish point identification of the nonparametric distribution of the random parameters. Our main identifying assumption is that all the observable covariates $(X, Z) = (X_1, X_2, Z_1, Z_2)$ are independent of the unobserved random vector $(B, \Delta) = (B_1, B_2, \Delta_1, \Delta_2)$, conditional on additional covariates W. We think of the system (1.1) as a system of simultaneous equations and note, as is well known in the literature, that the properties of the model change fundamentally with the sign of the interaction effects, see, e.g., Bresnahan and Reiss (1991), or Tamer (2003). Therefore, we focus largely on subcases. In particular, we start out with the case where, in every market we use for identification, the players behave as "strategic substitutes", which is central to the literature on market entry. In our setup, this means that there is always a negative externality from a player entering the market on the net utility of the other player, but to a varying degree across markets. We also cover the case of "strategic complements", where the other player's action positively affects the player's own utility, which is plausible, for example, in the joint work and retirement decision.

Our main result states that, in the case of strategic substitutes, the joint densities of $B + \Delta$ and B, respectively, are point-identified. The intuition behind this result is as follows. One can relate the joint characteristic function of B to the conditional entry probability of $(Y_1, Y_2) = (0, 0)$ and similarly that of $B + \Delta$ to the conditional probability of $(Y_1, Y_2) = (1, 1)$. The exogenous variation of the covariates (Z, X) then allows us to trace out the joint characteristic functions. To this end, the variation in the covariates has to be sufficiently large. Specifically, we require that the covariates $Z = (Z_1, Z_2)$ have joint full support. The support requirement on other covariates X depends on assumptions we are willing to place on the density of interest. If we assume that the characteristic function of the density is analytic, it suffices for X's support to contain a small open set. This requirement can be met in various applied examples, and hence we view it as the main empirically relevant requirement. Alternatively, if we assume that the density is entirely unrestricted, the covariates must have full support. While it highlights the required variation in the covariates for achieving point identification without any restriction on the density, this requirement is frequently unrealistic and also has limitations in terms of consistency with other assumptions. Once the characteristic functions of B and $B + \Delta$ are recovered, we may identify the density of Δ via deconvolution under the additional assumption that B and Δ are independent.

In either case, this result implies that the joint density of the interaction effects, f_Δ, is only set identified in general. However, under additional independence assumptions we obtain point identification of f_Δ and $f_{B\Delta}$ as well. In the case of strategic complements, we show that the joint characteristic function of $(B_1 + \Delta_1, B_2)$ can be related to the conditional probability of $(Y_1, Y_2) = (1, 0)$ and similarly that of $(B_1, B_2 + \Delta_2)$ is related to the conditional probability of $(Y_1, Y_2) = (0, 1)$. Therefore, these entry outcomes provide information that allows recovery of the marginal distribution of the interaction effects under additional independence assumptions. For example, Δ_1's distribution can be recovered from those of $B_1 + \Delta_1$ and B_1, however, the joint density of interaction effects f_Δ remains only partially identified in this scenario.

The identification principle put forward is constructive and can be employed to construct nonparametric sample counterpart estimators, whose analysis we defer to the companion paper. In addition to contributing to the abstract

[1] We distinguish notationally between a single covariate Z_j, whose coefficient we normalize to be unity almost surely, and the remaining vector of covariates X_j. As is implicit in the binary nature of the actions, because of the indicator function there is a choice of normalization. Throughout this paper, we assume that the sign of one of the original random coefficients be the same for the entire population. This allows us to normalize by this random coefficient and it is the corresponding variable which we denote by Z_j. In an earlier version of the paper (DHK, 2014), we show that the model is actually identified by (only) imposing a dual hemisphere normalization condition, but the economic benefits of this greater generality are minor.