Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

Is the diurnal pattern sufficient to explain intraday variation in volatility? A nonparametric assessment[☆]

Kim Christensen[a], Ulrich Hounyo[b,a,*], Mark Podolskij[c,a]

[a] Aarhus University, Department of Economics and Business Economics, CREATES, Fuglesangs Allé 4, 8210 Aarhus V, Denmark
[b] University at Albany –State University of New York, Department of Economics, 1400 Washington Avenue, Albany, NY 12222, United States
[c] Aarhus University, Department of Mathematics, Ny Munkegade 118, 8000 Aarhus C, Denmark

ARTICLE INFO

Article history:
Received 2 September 2016
Received in revised form 11 March 2018
Accepted 21 March 2018
Available online xxxx

JEL classification:
C10
C80

Keywords:
Bipower variation
Bootstrapping
Diurnal variation
High-frequency data
Microstructure noise
Pre-averaging
Time-varying volatility

ABSTRACT

In this paper, we propose a nonparametric way to test the hypothesis that time-variation in intraday volatility is caused solely by a deterministic and recurrent diurnal pattern. We assume that noisy high-frequency data from a discretely sampled jump–diffusion process are available. The test is then based on asset returns, which are deflated by the seasonal component and therefore homoskedastic under the null. To construct our test statistic, we extend the concept of pre-averaged bipower variation to a general Itô semimartingale setting via a truncation device. We prove a central limit theorem for this statistic and construct a positive semi-definite estimator of the asymptotic covariance matrix. The t-statistic (after pre-averaging and jump-truncation) diverges in the presence of stochastic volatility and has a standard normal distribution otherwise. We show that replacing the true diurnal factor with a model-free jump- and noise-robust estimator does not affect the asymptotic theory. A Monte Carlo simulation also shows this substitution has no discernable impact in finite samples. The test is, however, distorted by small infinite-activity price jumps. To improve inference, we propose a new bootstrap approach, which leads to almost correctly sized tests of the null hypothesis. We apply the developed framework to a large cross-section of equity high-frequency data and find that the diurnal pattern accounts for a rather significant fraction of intraday variation in volatility, but important sources of heteroskedasticity remain present in the data.

[☆] This paper was previously entitled “Testing for heteroscedasticity in jumpy and noisy high-frequency data: A resampling approach.” We appreciate the thoughtful feedback received from two anonymous referees, the associate editor, and Yacine Aït-Sahalia as part of the revision process, which helped to significantly improve the paper. We also thank for comments on an earlier version of this paper made by conference participants at the 10th Computational and Financial Econometrics (CFE) conference in Seville, Spain; the Vienna-Copenhagen (VieCo) 2017 conference in Vienna, Austria; the 10th Annual SoFiE meeting in New York, USA; the Canadian Econometric Study Group (CESG) 2017 meeting in Toronto, Canada; and at seminars in Aarhus, Aix-Marseille, Albany, California Polytechnic, Connecticut, Cornell, Erasmus (Rotterdam), Glasgow, Manchester, Nottingham and Queen’s University. The authors acknowledge research funds from the Danish Council for Independent Research (DFF-4182-00050). In addition, Podolskij thanks the Villum Foundation for the grant “Ambit fields: Probabilistic properties and statistical inference.” This work was also supported by CREATES, which is funded by the Danish National Research Foundation (DNRF78).

[*] Corresponding author.
E-mail address: khounyo@albany.edu (U. Hounyo).

https://doi.org/10.1016/j.jeconom.2018.03.016

Please cite this article in press as: Christensen K., et al., Is the diurnal pattern sufficient to explain intraday variation in volatility? A nonparametric assessment. Journal of Econometrics (2018), https://doi.org/10.1016/j.jeconom.2018.03.016.

1. Introduction

There is a widespread agreement in the literature that any dynamic model of volatility should – at a minimum – account for two distinct features in order to explain the formation of diffusive risk in financial markets. On the one hand, a mean-reverting but highly persistent stochastic component is needed at the *interday* horizon to capture volatility clustering (e.g., Fama, 1965; Mandelbrot, 1963). On the other, a pervasive diurnal effect is required as one of the most critical determinants to describe the recurrent behavior of *intraday* volatility. In stock markets, for example, there is a tendency for absolute (or squared) price changes during the course of a trading day to form a so-called "U"- or reverse "J"-shape with notably larger fluctuations near the opening and closing of the exchange than around lunch time (see, e.g., Harris, 1986; Wood et al., 1985, for early-stage documentation of this attribute).[1] In addition to these effects, volatility may exhibit large, sudden shifts around the release of important economic news, such as macroeconomic information (e.g., Andersen and Bollerslev, 1998).

A recent strand of work, fueled by access to high-frequency data and complimentary theory for model-free measurement of volatility, has taken a more detailed close-up of these components and largely confirmed their presence.[2] The diurnal U-shape, in particular, has emerged as a potent – if not predominant – source of within-day variation in volatility. It is therefore common to formulate *parametric* models of time-varying volatility targeted for high-frequency analysis (be it in continuous- or discrete-time) as a composition of a stochastic and deterministic process (with suitable restrictions imposed to ensure the parameters are separately identified). A standard approach is to assume that the stochastic process is constant within a day but is evolving randomly between them (thus enabling volatility clustering), while the deterministic part is a smooth periodic function that is allowed to change within the day but is otherwise time-invariant (thus capturing the diurnal effect), see, e.g., Andersen and Bollerslev (1997, 1998), Boudt et al. (2011), Engle and Sokalska (2012) and references therein.

Indeed, a major motivation behind the preferred use of realized measures of return variation that are temporally aggregated to the daily frequency is to avoid dealing with the diurnal effect, since it is widely believed to make them intrinsically robust against its presence. However, as stressed by Andersen et al. (2012) and Dette et al. (2016) diurnal effects inject a strong Jensen's inequality-type bias in some of these estimators; an effect that is reinforced and magnified with a high "volatility-of-stochastic volatility" (e.g., Christensen et al., 2014). This can, for instance, alter the finite sample properties of jump tests designed to operate at *either* the intraday *or* interday horizon (e.g., Andersen et al., 2007; Barndorff-Nielsen and Shephard, 2006; Lee and Mykland, 2008) and make them significantly leaned toward the alternative and cause spurious jump detection as result.[3] As such, further investigation of diurnal effects appears warranted.

In this paper, we develop a *nonparametric* framework to assess if diurnal effects can, in fact, explain all of the intraday variation in volatility, as stipulated by such a setup. A casual inspection of high-frequency data does not offer conclusive evidence about the validity of this conjecture. In concrete applications, inference is obscured by microstructure noise at the tick-by-tick frequency (e.g., Hansen and Lunde, 2006) and the existence of price jumps that are potentially very small and highly active (e.g., Aït-Sahalia and Jacod, 2012a).[4] Moreover, even if stochastic volatility is truly present, in practice its components may be so persistent that it is acceptable (and convenient) to regard it as absent on small time scales.[5]

Consistent with the above, we model the asset log-price as a general arbitrage-free Itô semimartingale, which is contaminated by microstructure noise. In this framework, the asymptotic theory is infill, i.e. the process is assumed to be observed on a fixed time interval with mesh tending to zero.

There are several existing tests of constant volatility available in the high-frequency volatility area (see, e.g., Dette et al., 2006; Dette and Podolskij, 2008; Vetter and Dette, 2012). To our knowledge, none allow for the joint disturbance of jumps and microstructure noise, nor do they directly study the extension to diurnal variation advocated here. We formulate a test on the back of log-returns that are only homoskedastic under the null, after they are filtered for diurnal effects. We then study a jump-robust version of the pre-averaged bipower variation, where we extend the bivariate central limit theorem of Podolskij and Vetter (2009a) to the jumpy setting (see Barndorff-Nielsen et al., 2008; Jacod et al., 2009; Zhang et al., 2005; Zhang, 2006, for further work on noise-robust volatility estimation). The test is constructed via the asymptotic distribution implied by a transformation of such statistics and an application of Cauchy–Schwarz for a particular – but standard – choice of the parameters. As an aside, we add that a slightly different configuration of our t-statistic (based on the comparison of suitably non-truncated and truncated statistics) can serve as a basis for a jump test, which is robust to diurnal effects, but we do not pursue this idea in the present paper.

---

[1] Moreover, diurnal effects are present in other financial variables, such as asset covariances and correlations, bid–ask spreads, trade durations, trading volume, and quote updates.

[2] A comprehensive list of papers in this field, including several reviews of the literature, is available at the webpage of the Oxford-Man Institute of Quantitative Finance's Realized Library: http://realized.oxford-man.ox.ac.uk/research/literature.

[3] The effects are deeply intertwined, however, because jumps can also induce substantial biases in and distort estimates of both integrated variance (e.g., Barndorff-Nielsen and Shephard, 2004; Christensen et al., 2014) and the diurnal pattern (e.g., Andersen et al., 2001; Boudt et al., 2011).

[4] In view of this, a related topic is whether a diffusive component is needed in the first place to represent risk formation in financial asset prices. While the prevailing evidence is slightly mixed, it appears largely affirmative (see, e.g., Aït-Sahalia and Jacod, 2012b; Kolokolov and Renò, 2017; Todorov and Tauchen, 2010).

[5] Of course, the locally constant approximation of stochastic volatility is one of the most heavily exploited in the analysis of high-frequency data, see, e.g., Mykland and Zhang (2009).

Please cite this article in press as: Christensen K., et al., Is the diurnal pattern sufficient to explain intraday variation in volatility? A nonparametric assessment. Journal of Econometrics (2018), https://doi.org/10.1016/j.jeconom.2018.03.016.