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Testing for jumps and jump intensity path dependence☆

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ABSTRACT

In this paper, we develop a "jump test" for the null hypothesis that the probability of a jump is zero, building on earlier work by Aït-Sahalia (2002). The test is based on realized third moments, and uses observations over an increasing time span. The test offers an alternative to standard finite time span tests, and is designed to detect jumps in the data generating process rather than detecting realized jumps over a fixed time span. More specifically, we make two contributions. First, we introduce our largely model free jump test for the null hypothesis of zero jump intensity. Second, under the maintained assumption of strictly positive jump intensity, we introduce two "self-excitement" tests for the null of constant jump intensity against the alternative of path dependent intensity. These tests have power against autocorrelation in the jump component, and are direct tests for Hawkes diffusions (see, e.g. Aït-Sahalia et al. (2015)). The limiting distributions of the proposed statistics are analyzed via use of a double asymptotic scheme, wherein the time span goes to infinity and the discrete interval approaches zero; and the distributions of the tests are normal and half normal. The results from a Monte Carlo study indicate that the tests have reasonable finite sample properties. An empirical illustration based on the analysis of 11 stock price series indicates the prevalence of jumps and self-excitation.

1. Introduction

Jump diffusions are widely used in the financial econometrics literature when analyzing returns or exchange rates, as discussed in Duffie et al. (2000), Singleton (2001), Andersen et al. (2002), Jiang and Knight (2002), Chacko and Viceira (2003) and Eraker et al. (2003), among others. In this context, various estimation techniques have been developed, and the common practice is to jointly estimate the parameters of both the continuous time and the jump components of models. Thus, parameters characterizing the drift, variance, jump intensity, and jump size probability density are jointly estimated. However, an obvious non-standard feature of this class of models is that the parameters characterizing the jump size density are not identified when the jump intensity is identically zero. This is an issue both when the intensity parameter is constant, as in standard stochastic volatility models with jumps (see, e.g. Andersen et al. (2002)) as well as when the intensity follows a diffusion process, as in the important case of the Hawkes diffusion models analyzed by Aït-Sahalia et al. (2015). If one estimates a jump diffusion model that contains a jump intensity parameter and if the population jump intensity happens to be zero, then a subset of the parameters in the model is not identified, which in turn precludes consistent estimation of other parameters (see Andrews and Cheng (2012)).

The above estimation problem serves to underscore the importance of pretesting for jumps. The first paper addressing the issue of discrimination between diffusion processes and jump processes was Aït-Sahalia (2002). He derived a set of necessary and sufficient conditions, based on the properties of the transition density, which have to be satisfied by any diffusion sampled at discrete times. Hence, he provided a criterion for checking whether there are jumps in the data generating process. Since then, there have been a large variety of tests for the null of no jumps versus the alternative of jumps. Tests include those based on the comparison of two realized volatility measures, one which is robust, and the other which is not robust to the presence of jumps (see, e.g. Barndorff-Nielsen et al. (2006) and Podolskij and Vetter (2009a)), tests based

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on a thresholding approach (see, e.g. Corsi et al. (2010), Lee and Mykland (2008), and Lee et al. (2013)), and tests based on power variation, as discussed in Aït-Sahalia and Jacod (2009). Such tests are consistent against realized jumps. One feature of these tests is that they are based on observations drawn on a given finite time span, and they can thus only detect whether jumps occurred during this given time span. While this is hardly a weakness of the existing tests, there are clearly situations for which interest lies in testing for the existence of jumps in the data generating process, or within a class of models. For example, this is the case if one is interested in using (transformations of) jump diffusion processes in a variety of valuation problems, such as option pricing and default modeling (see, e.g. Duffie et al. (2000)).

In this paper we make two contributions to the literature on jumps. First, we develop a "jump test" for the null hypothesis that the probability of a jump is zero, building on earlier work of Aït-Sahalia (2002). Second, under the maintained assumption of strictly positive jump intensity, we introduce a "self-excitement test" for the null of constant jump intensity against the alternative of path dependent intensity. This test has power against autocorrelation in the jump component, and is a direct test for Hawkes diffusions (see Aït-Sahalia et al. (2015)), in which jump intensity is modeled as a mean-reverting diffusion process. When the proposed tests are implemented prior to model specification, standard estimation of jump diffusions can be subsequently carried out, avoiding the identification problems discussed above. Recently, Boswijk et al. (2018) and Dungey et al. (2018) have suggested tests for self-excitation and mutual excitation in realized jumps. Our tests instead detect jump self-excitation in the data generating process.

Our jump test is based on realized third moments, or so-called tricity. Various realized tricity-type statistics over a finite time span have already been examined in the literature in order to: detect realized jumps, as in Jacod (2012); study the contribution of realized skewness when predicting the cross-section of equity returns, as in Amaya et al. (2015); and to test for the endogeneity of sampling times, as in Li et al. (2014). What distinguishes our tricity-type test from these is that it is analyzed using both in-fill and long-span asymptotics. The use of long-span asymptotics ensures that the suggested statistic has power against jump intensity rather than against realized jumps. Importantly, our test is also robust to the presence of leverage. The limiting behavior of the proposed statistic is readily analyzed via use of a double asymptotic scheme wherein the time span goes to infinity and the discrete interval approaches zero. Under the null hypothesis of zero intensity, the statistic has a normal limiting distribution. Under the alternative, it is necessary to distinguish between jumps with zero or non-zero third moment. In the latter case, the proposed test has a well defined Pitman drift and has power against $\sqrt{T}-$local alternatives, where T is the time span, in days. In the former case, the sample third moment approaches zero, but the probability order of the statistic is larger than that which obtains under the null, since the jump component does not contribute to the mean, while it does contribute to the variance. As the order of magnitude of the variance depends on whether the null hypothesis is true or not, we introduce a threshold estimator for the variance, which is consistent under the null of zero intensity, and bounded in probability under the alternative. Thus, inference can be performed via use of a simple t-statistic.

We suggest two versions of our self-excitement test, $S^{\beta}_{T^+,\Delta}$ and $\widetilde{S}^{\beta}_{T^+,\Delta}$, where $T^+ > T$, $T^+/T \to \infty$, and Δ is the discretization interval. The former is based on the autocorrelation function of returns, and the latter on the autocorrelation of squared returns. The advantage of the latter over the former is that it does not require non-zero mean jump size, while the former does.

In principle, one might consider testing for the null of zero intensity using a score, Wald or likelihood ratio test, based on discrete observations (see, e.g. Andrews (2001)). This approach requires treating jump size density parameters as nuisance parameters unidentified under the null, and requires correct specification of both the continuous and the jump components of the diffusion. Misspecification of one or both components will invalidate the test. Additionally, the likelihood function of a jump diffusion is not generally known in closed form, and therefore estimation (which is needed for test statistic construction) is usually based on either simulated GMM (see Duffie and Singleton (1993) and Andersen et al. (2002)), indirect inference (see Gourieroux et al. (1993) and Gallant and Tauchen (1996)), or nonparametric simulated maximum likelihood (see Fermanian and Salanié (2004) and Corradi and Swanson (2011)). However, it goes without saying that one cannot simulate a diffusion with a negative intensity parameter. This, in turn, precludes the existence of a quadratic approximation around the null parameters of the criterion function to be maximized (minimized). Given that the existence of such quadratic approximations is a necessary condition for estimation and inference about parameters on the boundary (see Andrews (1999, 2001), Beg et al. (2001), and Chapter 4 in Silvapulle and Sen (2011)), we cannot rely on simulation-based estimators when testing using standard score, Wald or likelihood ratio tests.

The finite sample behavior of the tests is studied in a series of Monte Carlo experiments. Since the tests are not robust to microstructure noise, one needs to choose a frequency for which the noise is not too binding. For this reason, in our Monte Carlo exercise, we set the discretization interval $\Delta = 1/78$ and $\Delta = 1/156$, corresponding to moderate frequencies. We also study test sensitivity to the presence of non-zero microstructure noise. The empirical size of the jump test is sensitive to the smallest values of T and Δ^{-1}, but performance is markedly better as their magnitude is increased. Moreover, the power is quite good across all parameterizations, even in the case of jumps with zero third moment. We then assess and compare the finite sample properties of the two self-excitation tests. For cases where jumps have non-zero mean, we find that $S^{\beta}_{T^+,\Delta}$ behaves better than $\widetilde{S}^{\beta}_{T^+,\Delta}$, in the sense of suffering from less size distortion. However, it is important to note that in our empirical analysis, all series examined are characterized by zero mean jumps, so that $S^{\beta}_{T^+,\Delta}$ is not informative, and $\widetilde{S}^{\beta}_{T^+,\Delta}$ does not suffer size distortion. As expected, both tests have good power as the level of path dependence increases. In our empirical illustration, we examine 11 U.S. stock price series. We find strong evidence of jumps and self-excitation, regardless of T, when analyzing data between 2003–2014.

The rest of the paper is organized as follows. Section 2 describes the set-up. Section 3 and Section 4 discuss the jump intensity and self-excitement tests, and derive their asymptotic properties, respectively. Section 5 reports the findings of a Monte Carlo study designed to examine the finite sample properties of the tests, Section 6 contains the results of an empirical illustration, and concluding remarks are gathered in Section 7. All proofs are collected in an Appendix.

2. Set-up

We consider stochastic volatility jump diffusions, with either constant or path dependent intensity. For $t \in \mathbb{R}^+$, consider

$$d\ln X_t = \mu dt + V_t^{1/2}\sqrt{1-\rho^2}dW_{1,t} + V_t^{1/2}\rho dW_{2,t} + Z_t dN_t, \tag{1}$$

and

$$dV_t = \mu(V_t,\theta)dt + g\left(V_t,\theta\right)dW_{2,t}, \tag{2}$$