Journal of Econometrics ▮ (▮▮▮▮) ▮▮▮–▮▮▮

Contents lists available at ScienceDirect

Journal of Econometrics

journal homepage: www.elsevier.com/locate/jeconom

A unified approach to volatility estimation in the presence of both rounding and random market microstructure noise

Yingying Li [a], Zhiyuan Zhang [b,*], Yichu Li [c]

[a] Hong Kong University of Science and Technology, Hong Kong Special Administrative Region
[b] School of Statistics and Management, Shanghai University of Finance and Economics, No. 777 Guoding Road, Shanghai 200433, China
[c] Investment Technology Group, NY, USA

ARTICLE INFO

Article history:
Received 13 January 2016
Received in revised form 22 November 2017
Accepted 26 November 2017
Available online xxxx

JEL classification:
G12
C22
C14

Keywords:
High-frequency data
Rounding errors
Market microstructure noise
Integrated volatility
Realized volatility

ABSTRACT

Widely used volatility estimation methods mainly consider one of the following two simple microstructure noise models: random additive noise on log prices, or pure rounding errors. Apparently in real data these two types of noise co-exist. In this paper, we discover a common feature of these two types of noise and propose a unified volatility estimation approach in the presence of both rounding and random noise. Our data-driven method enjoys superior properties in terms of bias and convergence rate. We establish feasible central limit theorems and show their superior performance via simulations. Empirical studies show clear advantages of our method when applied to both stocks data and currency exchange data.

1. Introduction

Volatility estimation based on high-frequency data has received great attention over the past decades. A main challenge is the presence of market microstructure noise. Sources of market microstructure noise include various frictions in financial markets such as bid–ask bounce, dealer's inventory control, specific trading mechanism of an exchange and so on. Rounding is also a main source of market microstructure noise because traded prices are all rounded to price grids.

Market microstructure noise accumulates at high frequencies and causes large biases in classical volatility estimators such as realized volatility (RV). This is most clearly seen in the volatility signature plot of Andersen et al. (2000), where RV is plotted against sampling frequencies and an increasing trend of RV as the sampling frequency increases (i.e., the sampling interval decreases) is observed for many stocks.

Great efforts have been made towards consistent and efficient estimators of (integrated) volatilities. Most of the currently widely used high-frequency volatility estimators (see, e.g., the two scales realized volatility (TSRV) by Zhang et al. (2005), the multi-scale realized volatility (MSRV) of Zhang (2006), the realized kernels (RK) by Barndorff-Nielsen et al. (2008), the pre-averaging volatility (PAV) estimator of Jacod et al. (2009), Podolskij and Vetter (2009) and the quasi-maximum likelihood estimator (QMLE) of Xiu, 2010) rely on the assumptions that the market microstructure noise is random and additive on log prices. More precisely, let $(X_t)_{0\le t\le 1}$ be the logarithm of latent price process that follows a semimartingale, then the observed log prices at discrete time points $0 \equiv t_0 < t_1 < \cdots < t_n \equiv 1$ are assumed to be of the form

$$X_{t_i} + \eta_{t_i},\ i = 0, 1, 2, \ldots, n,$$

where $(\eta_{t_i})_{i\ge 0}$ is usually assumed to be independently and identically[1] distributed, and independent of the $(X_t)_{0\le t\le 1}$ process, with $\mathrm{Var}(\eta_{t_i}) = O(1)$. See also Zhou (1996), Hansen and Lunde (2006), Aït-Sahalia et al. (2005) and Bandi and Russell (2006) for related works in this direction.

In Fig. 1, we plot volatility estimates given by the aforementioned random-noise-oriented methods against different sampling intervals for four stocks traded on the New York Stock Exchange

* Corresponding author.
E-mail address: zhang.zhiyuan@mail.shufe.edu.cn (Z. Zhang).

[1] A relaxed condition is assumed in Jacod et al. (2009), see also footnote 2.

https://doi.org/10.1016/j.jeconom.2017.11.006
0304-4076/

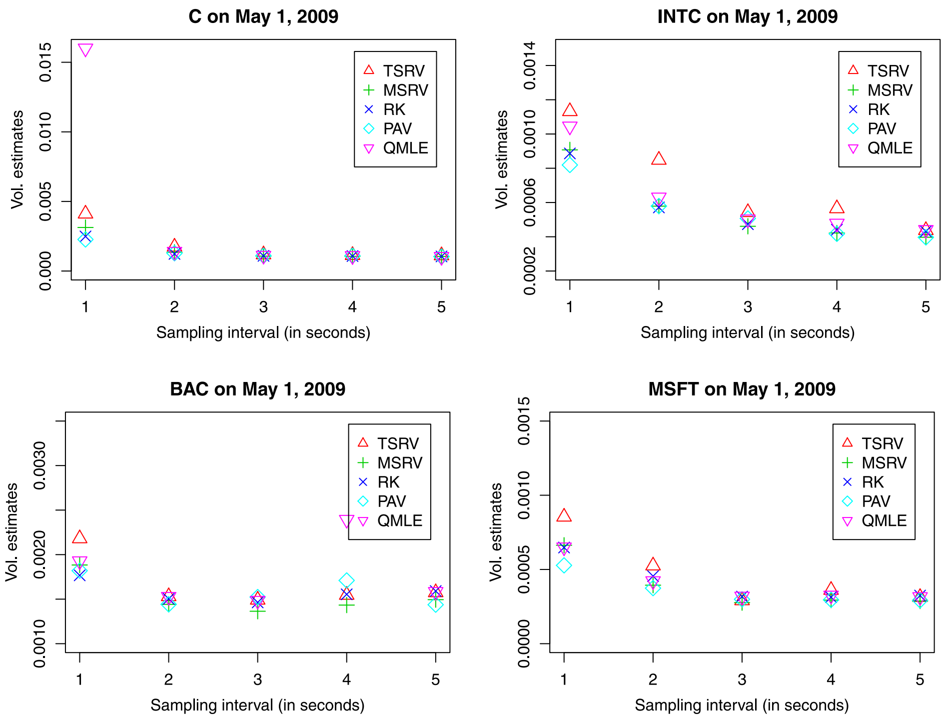

Fig. 1. Volatility estimates of five random-noise-oriented methods across different sampling intervals based on the second-by-second data of four stocks on May 1, 2009. The estimates given by these methods all diverge as the sampling interval becomes smaller, presenting a volatility signature plot pattern.

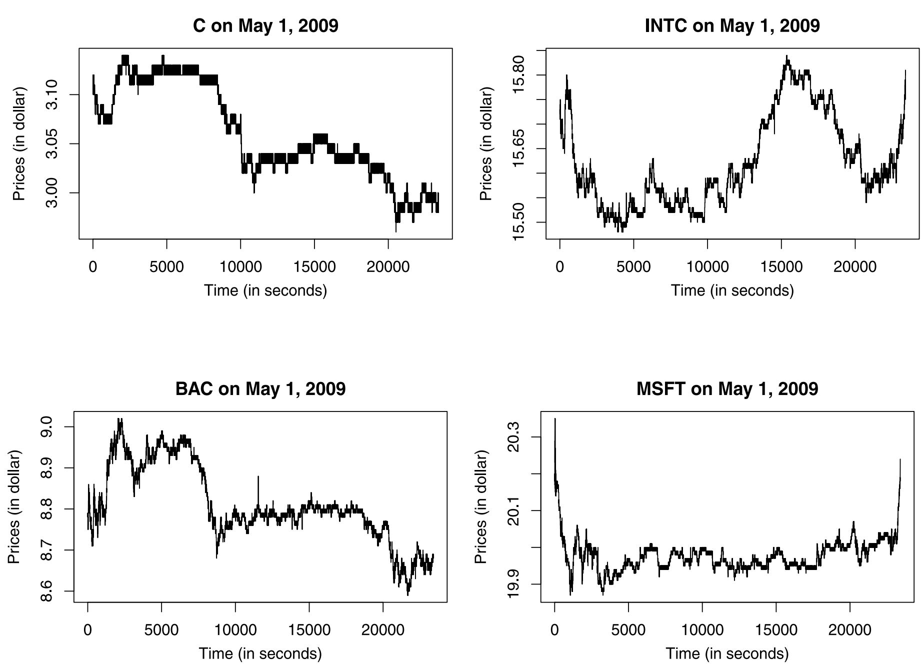

Fig. 2. Second-by-second trade prices of four stocks on May 1, 2009.

(NYSE) and the National Association of Securities Dealers Automated Quotation System (NASDAQ) on May 1, 2009 (Citigroup Inc. (NYSE: C), Intel Corporation (NASDAQ: INTC), Bank of America (NYSE: BAC) and Microsoft Corporation (NASDAQ: MSFT)). These stocks are all traded at high frequencies. Recall as we mentioned above, for many stocks there is an increasing trend of RV when the sampling interval becomes smaller, usually referred to as the "volatility signature plot" pattern. This pattern is undesirable because good volatility estimators should show a converging trend as sampling frequency goes higher rather than diverging. One would expect the random-noise-oriented methods mentioned above would no longer exhibit this signature plot pattern. However, in Fig. 1, we see that the widely used volatility estimators, TSRV, MSRV, RK, PAV and QMLE, still show volatility signature plot pattern.

What has gone wrong with these volatility estimators? The answer has to do with the fact that the observed high-frequency prices are also exposed to rounding errors. Li and Mykland (2007) show that ignoring rounding errors may lead to large biases in volatility estimation. We plot in Fig. 2 the second-by-second price

Please cite this article in press as: Li Y., et al., A unified approach to volatility estimation in the presence of both rounding and random market microstructure noise. Journal of Econometrics (2017), https://doi.org/10.1016/j.jeconom.2017.11.006.