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Cross-polarization properties of two Gaussian Schell-model beams through non-Kolmogorov turbulence

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ABSTRACT

General expressions are derived for the spectral degree of cross-polarization (SDCP) of a beam generated by the superposition of two Gaussian Schell-model (GSM) beams, which illuminated with the same Gaussian Schell-mode source propagating in non-Kolmogorov turbulent atmosphere by adopting beam cross-spectral density matrix and Young's interference theory. In particular, through numerical examples based on our analytical formal the SDCP of two GSM beams is analyzed. Detailed analysis demonstrate that the SDCP is closely to the spacing of two beams on source plane as well as the strength of the atmospheric turbulent, but the fractal constant α has no affect on the SDCP.

1. Introduction

The effects on the polarization features of the electromagnetic field across the output plane of a Young interferometer due to the correlations existing between the fields emerging from the two pinholes of the mask have been the subject of several recent works [1–9]. It has been shown in particular that the elements of the polarization matrix of the field across the output of the interferometer may differ from those of the field at the small apertures as the result of generalized interference law involving the second-order correlation between the field component at the pinholes [1,2]. In the theoretical analysis of such situation the beams are usually idealized by assuming them to be strictly monochromatic [3–4]. Recently other situations have been considered both within the formulation of the scalar theory [5] and the electromagnetic theory and two correlated stochastic electromagnetic beams [6–8]. Ding et al. [9] have studied the spectral partially and spectral switches of diffracted spatially and spectrally partially coherent pulsed beams in Young's interference experiment.

Recently the spectral degree of cross-polarization of pairs of points in a stochastic electromagnetic beam has been studied [10–14]. The spectral degree of cross-polarization of a stochastic electromagnetic beam like field on propagations through the turbulent atmosphere is analyzed [15]. Because the non-Kolmogorov turbulence spectrum model is more general than Kolmogorov spectrum so recently theoretical treatment and analytical solutions based on non-Kolmogorov turbulence have been proposed to describe optical wave statistics and beam parameters [16–22].

In the present paper we analyze the SDCP of light in the receiving pattern formed by the two beams propagating through the turbulent atmosphere. In Section 2, we give the cross-spectral density matrix in the turbulence atmosphere. The SDCP on the interference pattern is derived based on the introduced method of the cross-spectral density matrix in Section 3. The numerical calculation and analysis how the distance of two beams, the structure constant of the atmospheric turbulence and the fractal constant α affect the SDCP in Section 4. Finally, a brief conclusion is given in Section 5.

2. Cross-spectral density matrix of GSM beams through turbulent atmosphere

An opaque screen pierced with two identical, small apertures located at $Q_1(h)$ and $Q_2(-h)$ with position vectors h_1 and h_2 (Fig. 1), respectively, are illuminated with the same Gaussian Schell-mode source. Each small aperture thus becomes a secondary emitter with Gaussian Schell-mode beam labeled by Q_1 or Q_2. An observation plane is placed at z distance behind the opaque screen and parallel to the opaque screen [23]. The spacing between the small apertures is taken to be much smaller than the distance between the opaque screen and the observation plane. At the same time we assume that the angles of incidence and of diffraction at the small apertures are small.

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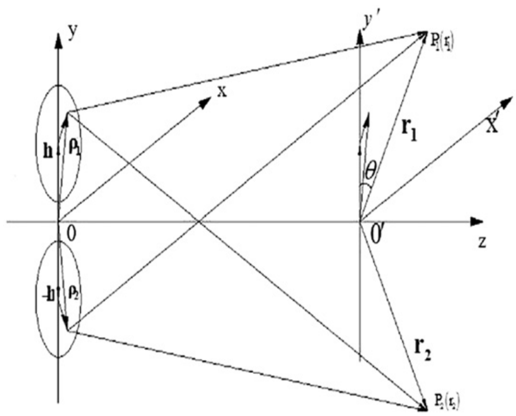

Fig. 1. Notation illustrated.

The second-order statistical properties of the field at small apertures may be characterized by the 2×2 electric cross-spectral density matrix defined by the formula [24]

$$\overleftrightarrow{W}^{(0)}(\rho_1,\rho_2,\omega)\equiv[W_{ij}^{(0)}(\rho_1,\rho_2,\omega)]=[\langle E_i^*(\rho_1,\omega)E_j(\rho_2,\omega)\rangle],\quad i=x,y,\quad j=x,y \tag{1}$$

where E_i and E_j are the Cartesian components of the GSM beam $E(\rho,\omega)$ at a point $(\rho,0)$ in the opaque screen plane; the asterisk denotes the complex conjugate and the angular brackets denote the average taken over a statistical ensemble of realizations of the electric field. Similarly, the second-order correlation properties of the GSM beams propagation through turbulent atmosphere and at point $P_1(r_1)$ and $P_2(r_2)$ in the plane of observation may be represented by the correlation matrix [25]

$$\overleftrightarrow{W}(r_1,r_2,\omega)=[W_{ij}(r_1,r_2,\omega)]=[\langle E_i^*(r_1,\omega)E_j(r_2,\omega)\rangle],\quad i=x,y,\quad j=x,y \tag{2}$$

where the components of the electric field at observation point $P_1(r_1)$ and $P_2(r_2)$ are given by [24]

$$E_i(r_\alpha,\omega)=-\frac{ik}{2\pi z}\left\{\iint E_i^{(0)}(\rho_1,\omega)\exp\left[\frac{ik}{2z}(r_\alpha-h-\rho_1)^2\right]\exp[\psi(\rho_1,r_\alpha,\omega)]d^2\rho_1\right.$$
$$\left.+\iint E_i^{(0)}(\rho_2,\omega)\exp\left[\frac{ik}{2z}(r_\alpha-h-\rho_2)^2\right]\exp[\psi(\rho_2,r_\alpha,\omega)]d^2\rho_2\right\}$$
$$i=x,y\quad \alpha=1,2 \tag{3}$$

where $k=2\pi/\lambda=\omega/c$ is the wave number associated with the frequency ω, λ is the wavelength and c is the speed of light in vacuum. $\psi(\rho_\beta,r_\alpha,\omega)$ is the random part of the complex phase of a spherical wave propagating through the turbulent atmosphere.

Substituting Eq. (3) in Eq.(2), the spectral correlation matrix of the electric field at a pair of points $P_1(r_1)$ and $P_2(r_2)$ in the observation plane can be represented by the elements of spectral correlation matrix $W^{(0)}$of the electric field at the small apertures as [26]

$$W_{ij}(r_1,r_2,\omega)=\langle E_i(r_1,\omega)E_j^*(r_2,\omega)\rangle$$
$$=\left(\frac{k}{2\pi z}\right)^2\left\{2\iint\iint W_{ij}^{(0)}(\rho_1,\tilde{\rho}_1,\omega)\right.$$
$$\times\exp\left\{\frac{ik}{2z}\left[(r_1-h-\rho_1)^2-(r_2-h-\tilde{\rho}_1)^2\right]\right\}$$
$$\times\langle\exp[\psi(\rho_1,r_1,\omega)+\psi^*(\tilde{\rho}_1,r_2,\omega)]\rangle d^2\rho_1 d^2\tilde{\rho}_1$$
$$+\iint\iint W_{ij}^{(0)}(\rho_1,\rho_2,\omega)$$
$$\times\exp\left\{\frac{ik}{2z}\left[(r_1-h-\rho_1)^2-(r_2-h-\rho_2)^2\right]\right\}$$
$$\times\langle\exp[\psi(\rho_1,r_1,\omega)+\psi^*(\rho_2,r_2,\omega)]\rangle d^2\rho_1 d^2\rho_2$$
$$+\iint\iint W_{ij}^{(0)}(\rho_1,\rho_2,\omega)$$
$$\times\exp\left\{\frac{ik}{2z}\left[(r_1-h-\rho_2)^2-(r_2-h-\rho_1)^2\right]\right\}$$
$$\left.\times\langle\exp[\psi(\rho_2,r_1,\omega)+\psi^*(\rho_1,r_2,\omega)]\rangle d^2\rho_1 d^2\rho_2\right\} \tag{4}$$

By use of the non-Kolmogorov power spectrum of atmospheric turbulence, the last term in Eq. (4) can be written as [22]

$$\langle\exp\left[\psi^*(\rho_\beta,r_\alpha,\omega)+\psi(\rho_{\beta'},r_{\alpha'},\omega)\right]\rangle$$
$$\cong\exp\left[-\frac{(\rho_\beta-\rho_{\beta'})^2+(\rho_\beta-\rho_{\beta'})(r_\alpha-r_{\alpha'})+(r_\alpha-r_{\alpha'})^2}{\rho_0^2}\right] \tag{5}$$

here

$$\rho_0=\left\{\frac{\pi^2\kappa^2 zA(\alpha)}{6(\alpha-2)}C_n^2\left[\kappa_m^{2-\alpha}\exp\left(\frac{\kappa_0^2}{\kappa_m^2}\right)(2\kappa_0^2-2\kappa_m^2+\alpha\kappa_m^2)\Gamma\left(2-\frac{\alpha}{2},\frac{\kappa_0^2}{\kappa_m^2}\right)-2\kappa_0^{4-\alpha}\right]\right\}\Bigg]^{-1/2},$$

C_n^2 is a generalized refractive-index structure parameter with units $m^{3-\alpha}$, $\kappa_0=2\pi/L_0$, L_0 is the out scale of turbulence, $\kappa_m=c(\alpha)/l_0$, $c(\alpha)=[\Gamma(5-(\alpha/2))A(\alpha)(2/3)\pi]^{1/\alpha-5}$, l_0 is the inner scale of turbulence, $A(\alpha)=(1/4\pi^2)\Gamma(\alpha-1)\cos(\alpha\pi/2)$, and $\Gamma(x)$ is the gamma function.

For a planar GSM source occupying a finite domain Σ, the elements of the electric cross-spectral density matrix (1) can be expressed in the form [27]

$$W_{ij}^{(0)}(\rho_1,\rho_2,\omega)=\sqrt{S_i^{(0)}(\rho_1,\omega)}\sqrt{S_j^{(0)}(\rho_2,\omega)}\mu_{ij}^{(0)}(\rho_2-\rho_1,\omega),\quad i=x,y\quad j=x,y \tag{6}$$

where $S_i^{(0)}(\rho,\omega)$ is the spectral density of the component E_i of electric field in the source plane [28]

$$S_i^{(0)}(\rho,\omega)=A_i^2(\omega)\exp(-\rho^2/2\sigma_i^2),\quad i=x,y \tag{7}$$

and $\mu_{ij}^{(0)}(\rho_2-\rho_1,\omega)$ denotes the spectral degree of coherence of the field across the source which is given by the expression

$$\mu_{ij}^{(0)}(\rho_2-\rho_1,\omega)=B_{ij}\exp(-(\rho_2-\rho_1)^2/2\delta_{ij}^2),\quad i=x,y,\quad j=x,y \tag{8}$$

in which the coefficients A_i, B_{ij}, σ_i and δ_{ij} are independent of position but they generally depend on the frequency ω. σ_i is the width of the source beam and δ_{ij} is the source correlation coefficient. Moreover, the coefficient B_{ij} satisfy relations $B_{ij}=1(i=j)|B_{ij}|\le 1(i\ne j)$ and $B_{ij}=B_{ji}^*$.

To simplify the subsequent analysis, we will take

$$\sigma_x=\sigma_y\equiv\sigma. \tag{9}$$

The cross-spectral density matrices of a GSM source are given by expression in the form

$$W_{ij}{}^{(0)}(\rho_1,\rho_2,\omega)=A_iA_jB_{ij}\exp\left(-\frac{(\rho_2-\rho_1)^2}{2\delta_{ij}{}^2}\right)\exp\left(-\frac{\rho_1{}^2+\rho_2{}^2}{4\sigma^2}\right),$$
$$i=x,y,\quad j=x,y \tag{10}$$

On substituting from Eqs. (5) and (10) into Eq. (4) and using a tensor method, we obtain the more general integral for the elements of cross-spectral density matrix in the output plane as follows

$$W_{ij}(r_1,r_2,\omega)=\left(\frac{k}{2\pi z}\right)^2\left\{2A_iA_jB_{ij}\iint\iint\exp\left(-\frac{(\rho_1-\tilde{\rho}_1)^2}{2\delta_{ij}{}^2}-\frac{\rho_1{}^2+\tilde{\rho}_1^2}{4\sigma^2}\right)\right.$$
$$\times\exp\left\{\frac{ik}{2z}\left[(r_1-h-\rho_1)^2-(r_1-h-\tilde{\rho}_1)^2\right]\right\}$$
$$\times\exp\left[-\frac{(\rho_1-\tilde{\rho}_1)^2+(\rho_1-\tilde{\rho}_1)(r_1-r_2)+(r_1-r_2)^2}{\rho_0^2}\right]$$