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Vortices from wavefront tilts

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We show evolution of optical vortices in the regions of a wavefront where circulating phase gradients are present. Two different regions in a wavefront are given different tilts and it is shown that phase singularities can evolve at discrete points along the line of phase discontinuity. The phase distribution and the gradient in the neighborhood of these points are studied. Using a spatial light modulator (SLM) we have experimentally demonstrated the vortex generation.

1. Introduction

Optical vortices or phase singularities are three dimensional threads of darkness where amplitude of the light field goes to zero and phase is undetermined. Vortices are naturally created and/or annihilated when a coherent beam of light passes through turbulent media [1]. Vortex infested wavefronts have helical shape. The growing range of applications of optical vortices calls for reliable methods of their generation. Several methods such as the use of spiral phase plate [2], helical mirror [3], deformable mirror [4] wedge plates [5–9], phase only diffractive optical element [10–12] and computer generated hologram [13] can be used for vortex generation.

In this paper we demonstrate a new method of vortex generation using only wavefront tilts. It is shown that if two different regions in a wavefront are given different tilts, at discrete points on the line of phase discontinuity, phase singularity can evolve upon propagation. Any two diametrically opposite points in the immediate neighborhood of these discrete points are out of phase and have opposite tilts.

The magnitudes of tilt required to generate vortices are very small. Hence a spatial light modulator (SLM) is used to provide opposite constant phase gradients (tilts), of small magnitude at two different regions of the wavefront. It is interesting to see that 2π helical phase variation can be realized by an element that can produce phase retardation less than 2π radians. The results presented here are significant towards understanding the evolution of vortices in diffracted optical fields. Vortex localization plays an important role in optical testing where vortices are used as phase markers [14]. The occurrence of small phase gradients in different regions of the wavefront is common in wave propagation. In a random phase distribution different regions of the same wavefront can have phase gradients different both in magnitude and direction. We develop the subject in the following manner. In Sections 2 and 3, we discuss about the nature of phase gradients of optical vortices and of pure wavefront tilts. In Section 4 phase distributions that can lead to vortex generation are discussed. In Section 5 the computational results of diffracted fields are presented. In Section 6 vortex detection methods are dealt with. Generation of vortices using SLM and their detection using Mach–Zehnder interferometer configuration are presented in Section 7. Results are discussed in Section 8.

2. Optical vortex

An isolated amplitude zero point on a wavefront around which the line integral satisfies the condition $\oint \nabla\phi \cdot dl = 2\pi m$ characterizes an optical vortex. Here $\nabla\phi$ is the phase gradient, ϕ is the phase distribution and m is an integer representing topological charge. In this paper we deal with vortices of charge ± 1. In the absence of vortex the above line integral is zero. Since we are concerned with singularity in the phase distribution, let us consider an optical vortex whose complex amplitude is given by [15]

$$U(x,y) = \frac{(x \pm jy)}{\sqrt{x^2+y^2}} = \exp\left[j\phi(x,y)\right] \quad (1)$$

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where $j=\sqrt{-1}$ and

$$\phi = Arg[x \pm jy]. \tag{2}$$

Only the transverse components of phase gradient in the vortex beam are considered. The transverse phase gradient $\nabla_{\perp}\phi$ of an optical vortex of charge +1, is given by

$$\nabla_{\perp}\phi = \frac{1}{r}\hat{\theta}. \tag{3}$$

Note that there is no radial component.

The Cartesian components of the phase gradient of vortex of charge +1can be found by writing the phase as

$$\phi(x,y) = \frac{1}{j}\ \ln\left[\frac{x+jy}{\sqrt{x^2+y^2}}\right] \tag{4}$$

$$\nabla_{\perp}\phi(x,y) = \left[\frac{x\hat{y}-y\hat{x}}{x^2+y^2}\right]. \tag{5}$$

The phase gradient in the x-direction (y-direction) is not a constant for a vortex. From Eq. (3), one can observe that as $r\rightarrow 0$, i.e. near the vortex core, $\nabla\phi\rightarrow\infty$. The phase gradient at any point depends on how far the point is located from the vortex core. At $r=r_0$, where r_0 is a constant, the magnitude of the phase gradient is constant for all values of θ and at $\theta=\theta_0$ and at $\theta=\theta_0+\pi$ these constant phase gradients point in opposite directions. This is depicted in Fig. 1.

3. Wavefront tilts

Let unit amplitude on-axis plane wave be incident on an optical element E as shown in Fig. 2. The wavefront tilt introduced by the optical element can be given by writing the transmittance

$$\exp(j2\pi(\alpha\, x+\beta\, y+\gamma\, z)) = \frac{e^{j(\overrightarrow{K}_T\cdot\overrightarrow{r})}}{e^{j(\overrightarrow{K}_I\cdot\overrightarrow{r})}} \tag{6}$$

where $\overrightarrow{K}_I$ and $\overrightarrow{K}_T$ are the wave vectors of the incident and transmitted waves, respectively, and $\overrightarrow{k}\,(\frac{2\pi}{\lambda})\hat{z}$.

$$\begin{aligned} \alpha\lambda &= \hat{K}_T\cdot\hat{x} \\ \beta\lambda &= \hat{K}_T\cdot\hat{y} \\ \gamma\lambda &= \hat{K}_T\cdot\hat{z} \end{aligned} \tag{7}$$

where $\hat{k}_T = k/|k|$ and $\alpha,\ \beta,$ and γ are spatial frequencies along x, y, and z directions, respectively. λ is the wavelength of light. Such a tilted wavefront can be generated using a mirror, or a wedge or a SLM. The phase gradient for a tilted wavefront is

$$\nabla\phi = 2\pi(\alpha\,\hat{x}+\beta\,\hat{y}+\gamma\,\hat{z}). \tag{8}$$

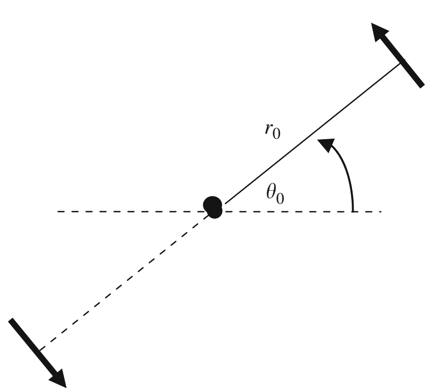

Fig. 1. Phase gradients at (r_0,θ_0) and $(r_0,\theta_0+\pi)$ in a vortex. In the beam cross-section the dark spot indicates the vortex core.

As before we consider only transverse components of phase gradient

$$\nabla_{\perp}\phi = 2\pi(\alpha\,\hat{x}+\beta\,\hat{y}). \tag{9}$$

Unlike an optical vortex, here the phase gradient is constant and does not depend on the location. Our aim is to generate vortex by bringing wavefronts with two different constant phase gradients near to each other and allow them to propagate.

4. Wavefront tilt configurations

We start our analysis by studying two wavefront tilt configurations shown in Fig. 3. These two configurations, which will be discussed later, are shown to generate vortices. In configuration I corresponding to Fig. 3(a) the complex amplitude in the

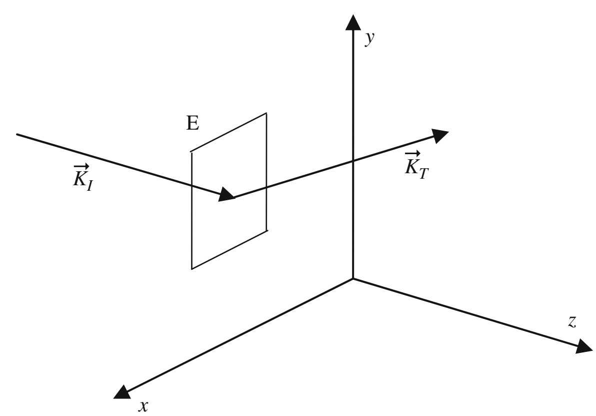

Fig. 2. Introduction of tilt by optical element E on the incident plane wave.

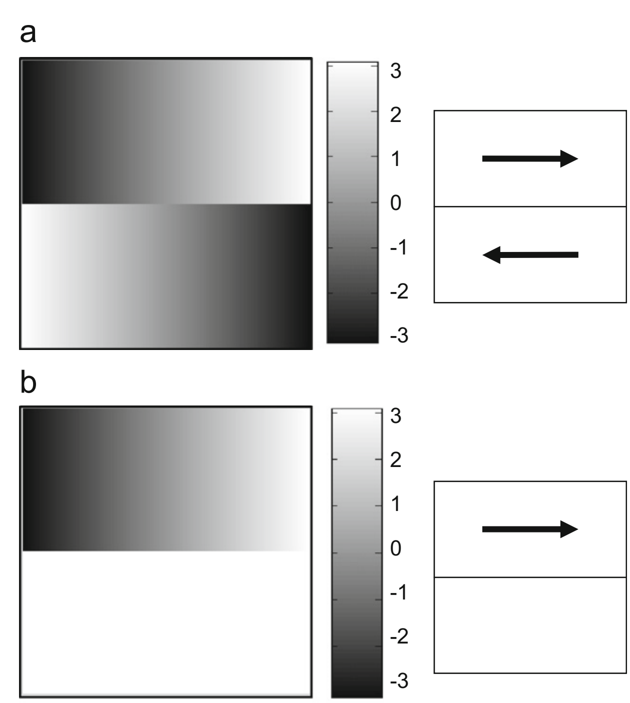

Fig. 3. (a) Configuration I, two linear phase variations in opposite directions in the upper and the lower region of the wavefront. (b) Configuration II, a linear phase variation in the upper region and a constant phase in the lower regions of the wavefront.