Journal of Economic Dynamics & Control 92 (2018) 153–182

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

# Portfolio selection with consumption ratcheting

Junkee Jeon[a], Hyeng Keun Koo[b,*], Yong Hyun Shin[c]

[a] Department of Mathematical Sciences, Seoul National University, Seoul 08826, Republic of Korea
[b] Department of Financial Engineering, Ajou University, 206 Worldcup-Ro, Yeongtong-gu, Suwon 16499, Republic of Korea
[c] Department of Mathematics, Sookmyung Women's University, Seoul 04310, Republic of Korea

ARTICLE INFO

*Article history:*
Received 13 October 2017
Revised 11 May 2018
Accepted 20 May 2018

*JEL classification:*
D11
E21
G11

*Keywords:*
Portfolio selection
Intolerance for decline in consumption
Risk attitude
Asset pricing

ABSTRACT

In this paper we study the portfolio selection problem of a finite-lived agent who does not tolerate a decline in standard of living. The preference can be regarded as exhibiting extreme-form of habit formation and also related to loss aversion in the prospect theory. We show that the agent's optimal portfolio exhibits a trend chasing behavior and the portfolio share of the risky asset fluctuates between 0 and the value of an unconstrained individual; the fluctuations do not attenuate as time gets near the end of the planning horizon. We also explore implications of the model for asset pricing and show that the model has a potential to match better the recent US data than traditional habit models. We provide a complete solution to the problem by considering a transformed problem, which is similar in its formal structure to an irreversible incremental investment problem and equivalent to an infinite series of optimal stopping problems.



## 1. Introduction

In this paper we study portfolio selection of an agent who does not tolerate a decline in standard of living. Dybvig (1995) studies an economic model in which an agent does not tolerate any decline in consumption and characterizes optimal consumption and portfolio strategies in an infinite horizon. He shows that the strategies exhibit ratcheting of consumption, which Duesenberry (1949) proposed as a realistic behavior of individual consumption (see also Brown, 1952). We study portfolio selection with a finite horizon and draw asset pricing implications.

In the real world ratcheting patterns are found in the design of pension products with guarantees, typically in that of variable annuities. Since 2002 a new form of guarantee, *Guaranteed Minimum Withdrawal Benefit (GMWB)* is offered with variable annuities (Bauer et al., 2008). With a GMWB option, a policyholder is entitled to withdraw a guaranteed minimum from her account. There are also ratcheting options with GMWB, in which the guaranteed minimum increases with an increase in the account value according to predefined rules (Kling et al., 2014). The ratchet options are consistent with the preference assumed in this paper.

The intolerance for decline in consumption puts a strong restriction on intertemporal consumption choice, which can be regarded as a strong form of habit formation (Dybvig, 1995). In the literature on habit formation an agent suffers a utility loss when current consumption falls short of the habit level which has been determined by her past consumption history

* Corresponding author.
*E-mail addresses:* hollman@snu.ac.kr (J. Jeon), hkoo@ajou.ac.kr (H.K. Koo), yhshin@sookmyung.ac.kr (Y.H. Shin).

https://doi.org/10.1016/j.jedc.2018.05.003
0165-1889/

(see e.g., Sundaresan, 1989, Constantinides, 1990). The utility loss from consuming less than the previously highest level of consumption (the habit level) is infinite for the preference considered in this paper.

The preference is also related to loss aversion in prospect theory. A loss averse agent evaluates utility by measuring gains or losses with respect to a reference point and is more sensitive to losses than to gains (Kahneman and Tversky, 1979, Tversky and Kahneman, 1991). An agent with intolerance for decline in standard of living sets a reference point at the previously highest level of consumption and exhibits an infinite sensitivity to a decline from the reference point.

The behavior of optimal portfolio exhibits features distinct from that of traditional portfolio selection models (e.g., Merton, 1971).[1] First, despite the assumption of a constant investment opportunity, the portfolio share of risky assets exhibits wide fluctuations in the ratcheting model, whereas it is constant in the traditional models for the benchmark case where the agent has constant relative risk aversion (Merton, 1971, Jagannathan and Kocherlakota, 1996). Second, optimal portfolio in the ratcheting model exhibits trend chasing (or positive feedback) behavior, which is absent in the traditional models.

As Dybvig (1995) has shown, the portfolio share of the risky asset exhibits fluctuations ranging from 0 to the optimal share of an unconstrained individual (i.e., an individual who does not have intolerance for consumption decline). We show that the fluctuations do not attenuate as time approaches the end of the planning horizon in a finite horizon model. Thus, the agent does not follow the popular investment advice to reduce exposure to risky assets as one gets older. Jagannathan and Kocherlakota (1996) evaluate the investment advice by using the standard economic reasoning and show that the advice is valid only when one has labor income which is not highly correlated with the returns on stocks. Viceira (2001) further shows with concrete calculation of optimal portfolio shares that the validity of the advice depends critically on the correlation of labor income growth with returns on stocks, magnitude of idiosyncratic risk in labor income growth, and risk aversion of the investor. There is no labor income in the model of this paper, and hence, the fact that the optimal policy does not follow the popular advice is consistent with these authors' critical evaluation of the popular advice. The optimal risky share fluctuates because the agent needs to keep wealth above the annuity value of current consumption. Thus, the portfolio choice strategy resembles a portfolio insurance strategy (see e.g., Basak, 1995; Basak, 2002). When the wealth level approaches the annuity value due to successive negative investment returns, the agent moves money from the risky asset to a riskless asset and the portfolio share approaches 0. When wealth accumulates to a sufficiently high level as a result of good investment returns, the agent increases consumption and the portfolio share approaches that of an unconstrained investor.

The trend chasing (or positive feedback) behavior, the second distinct feature of the optimal portfolio, means that the agent buys more shares when the price of the risky asset increases and sells shares when it decreases. The trend chasing behavior, in particular, implies that the agent appears to be less risk averse after a period of good investment returns and more risk averse after a period of bad (or negative) returns, similar to a model with loss aversion (e.g., Barberis et al., 2001).

The behavior of optimal portfolio implies that risk aversion appears to be generally higher than that of an unconstrained individual. Dybvig (1995) claims that the consumption ratcheting model can provide a resolution of the equity premium puzzle due to this feature of the model. We explore the asset pricing implications of the model. Following Constantinides (1990) and Marshall and Parekh (1999) we simulate optimal consumption processes of individuals and construct the aggregate consumption series by using a realistic set of parameter values. We show that the statistical properties of the aggregate consumption series are consistent with the those of the US data. In light of the critical evaluation of Otrok et al. (2002a) we show that the consumption ratcheting model has a better potential to explain the recent US data than the traditional non-extreme habit models.

We also make a technical contribution, providing a link between the consumption/investment problem and the irreversible investment problem involving real options (see e.g., Dixit and Pindyck, 1994). Dybvig (1995) has solved the problem by using the dynamic programming method. It is difficult to apply his method to the problem with a finite horizon, since one has to deal with a free boundary value problem associated with a non-linear Hamilton-Jacobi-Bellman (HJB) equation involving three variables, time, wealth and the past level of consumption.[2] We overcome the difficulty by considering a transformed problem, which is formulated in terms of the marginal utility (shadow price) of wealth process. The transformed problem involves only consumption decision, not portfolio choice. It involves the choice of a non-decreasing consumption process: at each instant an economic agent decides whether to increase consumption from the previous level, and if she chooses to do so, then decides the magnitude of the increase. Thus, the formal structure of the problem is similar to that of the irreversible investment problem considered by Pindyck (1988), in which a firm makes an irreversible decision on capacity expansion at each instant.[3] Pindyck (1988) considers a problem of choosing the time for making an incremental investment for each level of capacity greater than its initial value, which makes the problem equivalent to an infinite series

[1] The optimal portfolio share in the risky asset with ratcheting will always be positive.

[2] In an infinite horizon model the HJB equation does not involve the time variable, and hence, the problem becomes essentially one-dimensional since the value function is homogeneous with respect to wealth and the past level of consumption, if the felicity function has constant relative risk aversion as Dybvig (1995) has assumed. It is generally not very well-known how to solve a free boundary value problem or a variational inequality involving non-linear partial differential equations (see e.g., Soner and Shreve, 1989, Soner and Shreve, 1991).

[3] There is a significant difference between consumption ratcheting and irreversible investments. Ratcheting of consumption is an endogenous decision, while the irreversibility of investments is exogenous. Here we note the formal similarity between the two problems in the sense that both problems involve choice of non-decreasing processes.