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Dynamic analysis of discontinuous best response with innovation

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ABSTRACT

Starting from the static analysis in Eckert et al. (2017), we study a Cournot duopoly where firms can decide to incur fixed costs in activities that improve their competitiveness (i.e. product development or process innovation). Innovation costs generate discontinuities in the firms quantity best response functions and, in turn, a variety of equilibrium configurations, including multiple equilibria. We provide a dynamic global analysis of the equilibria and show the way in which firms' initial expectations regarding the rivals level of output are crucial in defining the configuration of the long run equilibrium.

1. Introduction

In many industries firms can increase their competitiveness investing in innovation to reduce costs (process innovation) or developing new products to better meet consumers' preferences and increase their willingness to pay (product innovation). Several contributions in the literature have studied the problem of strategic investments in innovation. One very fruitful approach used to understand the effects of market and technical uncertainty as well as strategic interaction is the stream of research on real options (see e.g. Dixit and Pindyck, 1994). Other contributions focus on innovation with multi-product competition and (production) capacity constraints (see Dawid et al., 2010; Dawid et al., 2013). A different branch of literature explores the effects of various behavioral rules behind firms' decision making. Specifically, several scholars have questioned the assumption of players' perfect knowledge on the structure of the game. Some have thus formulated dynamic economic models based on learning effects, misperception of market elements and expectations on rivals' actions. One of the major contributors in this respect has been Carl Chiarella, who touched upon all these elements in a number of his works (see, among many others, Agliari et al., 2006; Chiarella and He, 2003; Chiarella and Szidarovszky, 2004; Chiarella and Szidarovszky, 2005; Chiarella et al., 2002).

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The model studied in this paper is very closely related to the framework described in Eckert et al. (2017). The authors study a Cournot duopoly where firms can initially decide to invest in (product and/or process) innovation projects that will increase their competitiveness, before competing in the market selecting quantities. Innovation costs are assumed to be fixed. This is a sufficiently realistic assumption to describe those industries (e.g. pharmaceuticals and high-tech) experiencing large economies of scale and requiring significant sunk investments. The possibility of investing in product/process innovation generates discontinuities in the firms' quantity best response functions[1] and, for some parameter configurations, a multiplicity of stable Cournot–Nash equilibria, including asymmetric ones.[2] As highlighted by Eckert et al. (2017), the co-existence of multiple equilibria and the lack of obvious focal points may have important antitrust implications and pose the necessity to introduce dynamic adjustments to solve equilibrium selection problems. Eckert et al. (2017), however, consider only a static framework and focus their attention exclusively to the subset of parameters that guarantees the existence of asymmetric equilibria where only one firm invests in R&D. Our paper, instead, provides a complete characterization of the equilibria (including symmetric ones) in the static version. We then produce a dynamic global analysis with adaptive expectation learning and we show that the likelihood that a particular equilibrium configuration may be reached in the long run depends on the setup of the *learning process* and on the *initial expectations* that each player forms regarding the output level of the rival.[3] With this respect, we assume that players update their beliefs on rival's strategies according to adaptive expectations, which constitute a learning rule more sophisticated than static (or naive) expectations. As we point out in the paper, under this more realistic scheme, players can indeed learn to play symmetric as well as asymmetric Cournot–Nash equilibria. We also show that learning to play an equilibrium configuration is path-dependent. This calls for a full understanding of the dynamics of the system through its global properties. The importance of such global analysis in economics, and in particular in oligopoly models, has again been remarked several times also by Carl Chiarella (see for instance Bischi et al., 2004; Bischi et al., 2010).

From a mathematical point of view, the dynamics of the oligopoly can be studied through a bidimensional discontinuous piecewise-linear map, whose main peculiarity is the coexistence of stable fixed points and cycles. For this model, we characterize the shape of the basins of attraction under the different scenarios which depend on the cost of innovation and on expectations on rival's production. Thus, we study the local and global stability of each equilibrium, focusing on two particular cases. First, similar to Eckert et al. (2017), we consider the possibility that both firms can compete for the market incurring fixed costs to invest in product/process innovation (two-sided innovation). Second, we consider the possibility where only one firm can invest to boost competitiveness (one-sided innovation). This case, not considered in Eckert et al. (2017), is interesting because it allows us to study forms of natural product differentiation where, for example, the country of origin of a good may impose advantages/disadvantages on a brand (think of the importance of the "Made in" label for some brands).

The paper is organized as follows. Section two introduces the static model of two-sided innovation and describes the best response functions. Section three states the dynamic model and describes the different equilibria. Section four analyzes the main dynamic features of the system under naive and adaptive expectations. Section five proposes a variation of the previous model considering one-sided innovation. Section six concludes.

2. The static model

Here we develop the static model of the game. We shall study its dynamic version in Section 3.

Let us assume that the market is served by two *ex-ante* identical firms. Firm i, $i = 1, 2$, produces quantity q_i. Both firms can decide to invest in product/process innovation and incur cost $k_i > 0$. For the moment, let us focus our attention to the case in which $k_1 = k_2 = k$.

The inverse demand of a producer who does not innovate is

$$p_l = a_l - b(q_1 + q_2)$$

whereas, by investing in innovation, the inverse demand is

$$p_h = a_h - b(q_1 + q_2)$$

$b > 0$ and $a_h \geq a_l > 0$. a_h and a_l in particular model the degree of vertical differentiation, see Häckner (2000). Firms face constant marginal costs. In particular the marginal cost is c_h if the firm decides to invest in innovation and c_l otherwise. Assume that $0 < c_h \leq c_l$. Normalize $a_l - c_l = 1$ and define $a \equiv a_h - c_h$. Let us assume that $a \in (1, 2)$; $a > 1$ ensures that the investment k can be seen both as process and product innovation that, given the linearity of the demand and production costs, would produce the result of an increase in the margin between the reservation price and marginal costs; $a < 2$ ensures

[1] The feature of discontinuous best response functions has been already highlighted in the economic literature on entry deterrence. These contributions, see for example Bulow et al. (1985), Dixit (1979, 1980), Ware (1984) and Brander and Spencer (1983), consider in general the case of an incumbent that can decide to strategically invest in sunk costs, such as R&D, to deter the entry of a potential entrant.

[2] The possibility that competition among ex-ante identical firms may produce asymmetric equilibria has been already pointed out by Salant and Shaffer (1999). The authors, in particular, study the welfare implications of such asymmetric configurations and provide conditions under which asymmetric equilibria may be socially desirable. Relatedly, Zhao and Szidarovszky (2008) and Szidarovszky and Matsumoto (2016) fully characterize the Nash equilibria of an oligopoly with N firms endowed with discontinuous best responses.

[3] The techniques used here have been previously described in Bischi et al. (2010) and Bischi and Kopel (2001).