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Reviving Kalecki's business cycle model in a growth context

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ABSTRACT

In 1935 Kalecki formulated the first fully specified model of a macroeconomic dynamics in which he studied an endogenous and most elementary business cycle mechanism. To revive his insights, the present paper adapts his stationary economy to a growth context. Introducing a reasonable nonlinearity into the investment function, it then takes care that the model exhibits persistent cyclical behaviour that is characterized by a unique, globally attracting limit cycle. This feature is a robust property. A calibration of the numerical parameters achieves desired values for the cycle period as well as the amplitudes of the output-capital ratio and the capital growth rate. It is moreover demonstrated that a one-time shock to the dynamics can easily have long-lasting effects on the amplitudes. Despite common misgivings about delay differential equations (which here result from Kalecki's implementation lag), the analysis can be conducted with a limited mathematical effort.

1. Introduction

As early as 1935, Michał Kalecki advanced the first model of a macroeconomic dynamics, formulated in a precise and rigorous way that also meets mathematical standards. On the one hand, he anticipated the idea of the multiplier-accelerator that later became famous with the names of Samuelson (1939) and Hicks (1950). In addition, he incorporated as a phenomenon of empirical relevance an implementation lag (or "gestation period"), which is the nonnegligible amount of time that elapses from making an investment decision until the corresponding productive capacity is finally in place.

While it is true that in some versions of the discrete-time Samuelson-Hicks model family the lags may also be interpreted as taking account of such a gestation period, this is usually not an issue of any interest. By contrast, the implementation lag finds a clear expression in Kalecki's model, which is set up in continuous time with an explicit fixed delay in capital formation. This allows Kalecki to highlight the role that this feature plays for generating endogenous cyclical behaviour. In particular, without this lag the motions would be monotonic. As a matter of fact, the model works out the most elementary foundations for a theory of the business cycle.

Kalecki even takes one step further and in an ingenious way derives from German and American statistical data concrete numerical values for the parameters in his model. With the aid of a mathematical analysis he is then able to conclude that the oscillations in his model can indeed be viewed as business cycles, with a reasonable period of ten years. Hence

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Kalecki also anticipated what only fifty years later, with the advent of the Real Business Cycle school, became known and appreciated as numerical calibration.

The idea of an implementation lag was later taken up by Goodwin (1951) in his nonlinear accelerator model (likewise formulated in continuous time). Here cycles are already generated by his version of an accelerator mechanism and the nonlinearity built into it; the implementation lag is added subsequently "in order to come close to reality" (Goodwin, 1951, p. 11).[1] After that, however, this line of macroeconomic research was not further developed. Thus, Kalecki's model seems today not much more than a historical footnote.

The main reason for neglecting Kalecki's approach to the business cycle is certainly that the delay differential equation which he obtains is more complex to analyse than ordinary differential equations. Many theorists might be deterred by the mathematics they expect they have to master when they turn to Kalecki's concept of an implementation lag.[2] This is a pity in two respects. First, as just indicated, Kalecki reveals a most elementary mechanism to generate endogenous business cycles, historically as well as logically prior to the models advanced by Samuelson/Hicks, Kaldor, Metzler, Goodwin, which proved more influential in heterodox theory. A second point is that the role of mathematics in dealing with delays in continuous-time modelling is not as crucial as it may appear at first sight. Apart from some basic facts, of course, mathematics is actually not necessarily needed—and with growing complexity soon not even available any longer. As a rule, every slightly ambitious model will have to be studied on the computer anyway. The skills required from a researcher are then different from mathematical expertise: here one has to be able to simulate the dynamics and on this basis conduct numerical experiments in systematic and meaningful ways.

Also when other themes in macroeconomics are considered, it has to be said that the concept of time delays in a continuous-time framework ekes out a shadowy existence. On the whole, there are just a few specialists who are concerned with them.[3] A further reason for their lacking attraction is the fact that virtually all of these models take no great interest in economic theory but focus on mathematical (plus numerical) issues. As a consequence, the models are economically rather simple. In particular, except for applications to the neoclassical Solow growth model, this work restricts itself to stationary economies.[4] This state is somewhat unsatisfactory because one feels that actually it should not be too hard to come up with economically more meaningful growth versions, which nonetheless would give rise to very similar dynamic properties.

This is the point where the present paper sets in. In reconsidering Kalecki's model, it makes a straightforward proposal to translate it into a growth context. Formally, a nonlinearity comes into being in this way, but a basic property of the originally linear dynamics persists, that is, except for a fluke, the oscillations continue to either explode or die out. Therefore, in order to obtain robust fluctuations that are self-sustaining, a reasonable and convenient nonlinearity is subsequently introduced that smoothly bounds investment from above and below. It will be seen that over a wide range of parameters this generates a unique limit cycle that is globally attracting, a feature that permits us to refer to *the* business cycle of the economy. We also follow Kalecki in his calibration endeavour. Numerical parameter values will be given that do not only achieve a desired cycle period, but also desired amplitudes of the output-capital ratio and the capital growth rate.

Returning to the possible fears of getting lost in too technical mathematics, local stability of the equilibrium is, of course, the first issue an analysis has to deal with. In principle, one could do without mathematics and fix all numerical parameters in the model except two, lay a grid over the plane of these two, simulate the model for each such pair, and record whether the dynamics converges or diverges. In this way a stability frontier is obtained that separates the regions of stability and instability. By varying a third parameter one could furthermore study the resulting shifts of the frontier. Generally, such a procedure would indeed be the method of choice. This notwithstanding, in the present case a mathematical stability theorem still exists that is not too difficult to apply. While it is a bit more involved than just checking the trace and determinant of a 2×2 Jacobian matrix, the computer does not mind the arising square root and arc cosine in the stability condition. However, only a slight extension of the present model would probably deny us this convenience and we would have to resort to the numerical brute force alternative.

The remainder of the paper is organized as follows. The next section recapitulates Kalecki's model and puts forward our growth version. Section 3 studies the conditions for local stability and cyclical behaviour. Section 4 introduces the nonlinearity in the investment function and is then devoted to the calibration of the global dynamics. Section 5 discusses the long-lasting effects of a possible shock to it, and Section 6 concludes.

[1] For a modern mathematical analysis of the role of the length of the implementation lag in Goodwin (1951) model, see Matsumoto and Suzuki (2008). Because this issue of JEDC is devoted to the memory of Carl Chiarella, his alternative treatment of Goodwin's model in Chiarella (1990, Chapters 2 and 3) may be mentioned. He finds it convenient to specify the delay in investment as a distributed lag with exponential decay. This allows him to transform the dynamics into an ordinary nonlinear differential equation of second order with limit cycle behaviour. He is thus furthermore able to derive qualitative information about the cycle by the method of averaging.

[2] Admittedly, most contributions to differential equations in economic models with delays are fairly technical indeed.

[3] Most prominently, these are A. Matsumoto, F. Szidarovszky, M. Szydłowski, A. Krawiec. Generally, analytical results may not be quite as limited as it appears to the outsider. As a referee has pointed out, stimulating in this respect may be Kuang (1993) and He et al. (2009).

[4] The only two exceptions the author knows of are Jarsulic (1993) and Yoshida and Asada (2007). For short it may be said, however, that they get around the problem arising from growth in a somewhat ad-hoc manner.

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