Journal of Economic Dynamics & Control 000 (2018) 1–18

Contents lists available at ScienceDirect

Journal of Economic Dynamics & Control

journal homepage: www.elsevier.com/locate/jedc

Estimation of agent-based models using sequential Monte Carlo methods

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ARTICLE INFO

Article history:
Received 13 April 2017
Revised 9 November 2017
Accepted 13 January 2018
Available online xxx

JEL classification:
G12
C15
C58

Keywords:
Agent-based models
Estimation
Markov chain Monte Carlo
Particle filter

ABSTRACT

Estimation of agent-based models is currently an intense area of research. Recent contributions have to a large extent resorted to simulation-based methods mostly using some form of simulated method of moments estimation (SMM). There is, however, an entire branch of statistical methods that should appear promising, but has to our knowledge never been applied so far to estimate agent-based models in economics and finance: Markov chain Monte Carlo methods designed for state space models or models with latent variables. This latter class of models seems particularly relevant as agent-based models typically consist of some latent and some observable variables since not all the characteristics of agents would mostly be observable. Indeed, one might often not only be interested in estimating the parameters of a model, but also to infer the time development of some latent variable. However, agent-based models when interpreted as latent variable models would be typically characterized by non-linear dynamics and non-Gaussian fluctuations and, thus, would require a computational approach to statistical inference. Here we resort to Sequential Monte Carlo (SMC) estimation based on a particle filter. This approach is used here to numerically approximate the conditional densities that enter into the likelihood function of the problem. With this approximation we simultaneously obtain parameter estimates and filtered state probabilities for the unobservable variable(s) that drive(s) the dynamics of the observable time series. In our examples, the observable series will be asset returns (or prices) while the unobservable variables will be some measure of agents' aggregate sentiment. We apply SMC to two selected agent-based models of speculative dynamics with somewhat different flavor. The empirical application to a selection of financial data includes an explicit comparison of the goodness-of-fit of both models.

1. Introduction

Validation of agent-based models is currently an intense area of research. Various disciplines have developed distributed computational models of interacting agents that due to their inherent complexity appear to pose non-trivial demands when it comes to estimation of their parameters. Examples include the estimation of behavioral parameters in transportation networks (Molina et al., 2005), ecological models (Golightly and Wilkinson, 2011), sociological models of network formation (Snijders, 1996) and behavioral models of speculative behavior in financial markets. The latter has become a particularly burgeoning area of research. Recent contributions in this area have been surveyed by Lux and Zwinkels (2017). Various approaches can be found in this emerging literature: A large body of literature is concerned with models with a limited

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https://doi.org/10.1016/j.jedc.2018.01.021
0165-1889/

Please cite this article as: T. Lux, Estimation of agent-based models using sequential Monte Carlo methods, Journal of Economic Dynamics & Control (2018), https://doi.org/10.1016/j.jedc.2018.01.021

number of different strategies (mostly only two) whose dynamic evolution can be formalized by a system of typically highly nonlinear difference or differential equations in the limit of an infinite population of agents. This literature has been initiated by Day and Huang (1990), Chiarella (1992), and Brock and Hommes (1997, 1998), among others. Models of this class have been estimated by a variety of algorithms: for instance, Boswijk et al. (2007) estimate the regime-switching model of Brock and Hommes using a nonlinear least-squares approach, while Amilon (2008) estimates the same model using the efficient method of moments based on an auxiliary ARCH model.

More recently, Recchioni et al. (2015) and Lamperti et al. (2017) have resorted to calibration methods rather than econometric estimation to fit the same model to data. This alternative approach is motivated by the observation that even the relatively simple agent-based models of this class come along with a large number of parameters. When embedding a simulation model with many parameters into the loop of an iterative algorithm for parameter optimization, computational demands tend to increase easily beyond feasible boundaries. These computational limitations of 'traditional' statistical approaches have led researchers to select 'parameters of interest' for estimation and fix a priori the remaining ones. The calibration methods of Recchioni et al. (2015) and Lamperti et al. (2017) try to cope with the computational burden by using more efficient approaches: Recchioni et al. (2015) select parameters based on a constraint optimization using the squared residuals of the fit of the raw price data as the underlying loss function, while Lamperti et al. (2017) apply a machine learning algorithm based on a surrogate model that facilitates the exploration of new parameter values in successive rounds of the learning algorithm.

Other recent contributions have cut down on the number of parameters and have applied the simulated method of moments as a most generally applicable tool to estimate parameters, cf. Franke and Westerhoff (2011, 2012, 2016). Kukačka and Barunik (2017) have applied a simulated maximum likelihood approach, while Barde (2016) applies an information-theoretic criterion to calibrate and compare three closely related agent-based models that all can be expressed in a structural form as systems of difference equations.

Recent literature has also embarked on estimation of agent-based models not cast in structural form. Grazzini and Richiardi (2015) and Jang (2015) were probably the first who studied the estimation of a model in which a full set of agents is simulated. The underlying models are very simple ones with only few parameters: Grazzini and Richiardi illustrate minimum distance estimation of agent-based models by example of two simple models: an order book model with adaptive adjustment of limit prices, and a model of adaptation of a new product in a finite population of agents. Following Alfarano et al. (2008), Jang embeds Kirman's model of herding among speculators (Kirman, 1993) into a simple asset pricing framework. While this model can, in principle, be approximated by a stochastic difference equation (it is used by Barde, 2016, in this format), Jang (2015) uses microscopic simulations of the evolution of the strategies of the market participants. He shows that the objective function used in a simulated method of moments (SMM) algorithm is generically non-smooth, and often exhibits multiple equilibria as well as relatively flat areas over wide ranges of the parameter space. Exploring this issue further, Chen and Lux (2017) and Ghonghadze and Lux (2016) show that execution of a single SMM estimation from an arbitrary set of initial conditions could lead to virtually arbitrary results. On the base of extensive Monte Carlo simulations, they recommend to start the estimation with a comprehensive grid search followed by the application of a gradient-free optimization method for a sample of best fitting grid points. They also note strong correlations between the estimates of the three parameters of this model, a long preasymptotic range of the estimates before converging to $T^{1/2}$ consistency, and severe size distortions of the goodness-of-fit test based on the over-restrictions of the SMM algorithm. In conclusion, it appears that while SMM is a perfectly general approach to estimate full-fletched agent-based models, it suffers from important limitations: Firstly, finding the minimum of the SMM objective function can be computationally very challenging given the inconvenient properties mentioned above. Second, to obtain a decent signal-to-noise ratio of the estimated parameters, very large data sets (of order 10^5 to 10^6 in the above papers) would be needed. The latter problem reflects the generally lower efficiency of estimators based upon few moments compared to estimation methods that use more information. Chen and Lux (2017) argue that these problems might extend beyond the particular model explored in their paper, and might have as their principal cause the limited range of moments available for univariate financial data (basically measures of their heavy tails and clustering of volatility).

In view of this unsatisfactory scenario, the present paper aims at expanding the tool box available for the estimation of agent-based models into a new direction. While extant research has used a number of diverse estimation methods, there is, still an entire branch of statistical methods that should appear promising, but has to our knowledge never been applied so far to estimate agent-based models in economics and finance. This branch of statistical methods is Sequential Monte Carlo designed for state space models or more general models with latent variables. This class of models seems particularly relevant as agent-based models typically would consist of some latent and some observable variables since not all the characteristics of agents would mostly be observable. Hence, such models can be very naturally classified as hidden or latent variable models. Indeed, one might often not only be interested in estimating the parameters of a model, but one would also be interested to infer the time development of some latent variable from the observable ones. Extracting information on unobservable variables is a classical *filtering* problem for which state-space models are prototypical examples. The classical approach to inference in state-space models is the Kalman filter which is the optimal solution to the tasks of state filtering and parameter estimation for linear systems with Gaussian noises in both the dynamic laws governing the latent and observable variables (Grewal and Andrews, 2008).

However, agent-based models when interpreted as latent variable models would be typically characterized by non-linear dynamics and non-Gaussian fluctuations and, thus, would require a more general approach to statistical inference. Here

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