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Continuous-time smooth ambiguity preferences

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ABSTRACT

This study extends the smooth ambiguity preferences model proposed by Klibanoff et al. (2005) to a continuous-time dynamic setting. It is known that the original smooth ambiguity preferences converge to the subjective expected utility as the time interval shortens so that decision makers do not exhibit any ambiguity-sensitive behavior in the continuous-time limit. Accordingly, this study proposes an alternative model of these preferences that interchanges the role of the second-order utility function with that of the second-order probability to prevent the smooth ambiguity attitude of decision maker from evaporating in the continuous-time limit. By utilizing the utility convergence results established by Kraft and Seifried (2014), our model is eventually represented by the stochastic differential utility with distorted beliefs so that most existing techniques in economics and financial studies can be made applicable together with these distorted beliefs. We give an asset-pricing example to demonstrate the applicability of our model.

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1. Introduction

For a long time, the subjective expected utility (SEU) has been the standard theory of decision-making under uncertainty. According to this theory, even when a decision maker (DM) does not know the objective probability for uncertain payoffs, he/she makes a decision based on a unique subjective probability measure. Therefore, the DM should be indifferent between a *risky* bet where the objective probability measure is known and an *ambiguous* bet where that measure is unknown.

However, Ellsberg (1961) challenges the SEU as a normative theory of decision-making. Based on a well-designed thought experiment, he argues that DMs would be ambiguity-averse in most cases; that is, most DMs would prefer a risky bet over an ambiguous bet. Subsequent experimental studies have supported Ellsberg's conjecture, and many theoretical models incorporating DM's attitude toward ambiguity as well as risk have been developed.

Among these theories, Klibanoff et al. (2005), henceforth denoted by KMM, develop a so-called smooth ambiguity preferences model.[1] Assuming multiple probabilities for relevant payoffs, they model the DM's uncertainty attitude through double expected utilities. In the KMM model, the DM subjectively estimates the relative likelihood of each probability measure being the true measure. These likelihoods are called the second-order probability, whereas each probability measure for the relevant payoffs is called the first-order probability. In the first stage of the KMM model, the DM calculates his/her expectations of the first utility function of the relevant payoffs under each first-order probability. In the second stage, the

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[1] Other notable ambiguity-sensitive preferences include the maxmin utility of Gilboa and Schmeidler (1989), the Choquet expected utility of Schmeidler (1989), the multiplier preferences of Hansen et al. (1999) and Hansen and Sargent (2001), and the variational preferences of Maccheroni et al. (2006).

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DM employs the second-order probability to calculate his/her expectation of the second utility function of the first-stage expected utilities. Klibanoff et al. (2009) extended their KMM model to a dynamic model under a discrete-time setting.

However, Skiadas (2013) has shown that the KMM preferences converge to the SEU in the continuous-time limit. An implication for this result is as follows: as the Arrow-Pratt risk premium is proportional to the variance of risky payoffs, the ambiguity premium implied by the KMM preferences is proportional to the variance of the first stage expected utilities. In the continuous-time limit, the difference in these expected utilities is expressed as the difference in the drift rate under the Brownian uncertainty and the difference in intensity rate under the Poisson uncertainty, both of which are proportional to the time interval. Therefore, the variance of the first stage expected utilities is proportional to the square of the time interval, which becomes negligible in the continuous-time limit.

Given that many powerful techniques are aligned with continuous-time modeling, this study combines those with the KMM preferences. For this purpose, we apply Yaari (1987) *dual theory* to the KMM model and interchange the role of the second-order utility in the original KMM preferences with that of the second-order probability. Using this trick, we prevent the DM's ambiguity attitude from evaporating in the continuous-time limit. While this study concentrates on the Brownian uncertainty case, our model could also incorporate Poisson jump processes.

In their seminal paper, Kraft and Seifried (2014) present the rigorous proof of connection between the discrete-time recursive utility developed by Kreps and Porteus (1978) and the continuous-time stochastic differential utility (SDU) proposed by Duffie and Epstein (1992). By utilizing Kraft and Seifried (2014) convergence results, the DM's preferences in our model are eventually represented by the SDU with distorted beliefs, which enables us to handle the DM's ambiguity aversion very easily through the distorted beliefs.[2] In addition, experimental researchers could extract the degree of subjects' ambiguity aversion much easier through their subjective probabilities than through their utility functions. In these respects, our model could contribute to both theoretical and empirical literature.

Gindrat and Lefoll (2011) also extend the KMM preferences to the continuous-time dynamic setting. They achieve this extension by assuming that ambiguity aversion coefficient essentially becomes infinity as the time interval shortens to zero. In contrast, the degree of the DM's ambiguity aversion in our model can be well defined independently from the time interval. We believe that this difference is critical because attitude toward ambiguity as well as risk should be treated as the DM's intrinsic nature. Therefore, a model that enables us to conduct a frequency-independent comparative analysis of the DM's ambiguity attitude would be more reasonable.

We apply our model to asset pricing and calibrate our model under a setting that is the continuous-time extension of Ju and Miao (2012) model. Under the discrete-time setting, Ju and Miao (2012) indicate that the KMM preferences help resolve many asset pricing puzzles, including the equity premium puzzle, the equity-volatility puzzle, and the risk-free rate puzzle. Our calibration results reveal that our model takes over these properties, which suggests that our model can be actually regarded as a continuous-time analogue of the original KMM preferences. Furthermore, owing to the continuous-time modeling, we can explicitly decompose the equity premium into three parts: the risk premium, the premium for late resolution of uncertainty, and the ambiguity premium.

The remainder of this study is constructed as follows. Section 2 describes the basic setting considered in this study. Section 3 derives the certainty equivalent (CE) approximation of smooth ambiguity preferences in the discrete-time setting. Section 4 maps the CE approximation to the SDU by utilizing Kraft and Seifried's (2014) convergence results. Section 5 applies our model to asset pricing. Section 6 concludes the study.

2. Probability setting and consumption plan

This study concentrates on the Brownian uncertainty case. We consider a k-dimensional standard Brownian motion (SBM), $B_t = (B_t^1, B_t^2, \dots, B_t^k)^\top$, defined on a reference probability space $(\Omega, \mathscr{F}, \mathbb{P})$. The filtration $\{\mathscr{F}_t\}_{t=0}^T$ is generated by B_t and $\mathbb{P}$-null sets of $\mathscr{F} = \mathscr{F}_T$, where T is the terminal date.

With ambiguity, the DM cannot identify the true probability measure on $(\Omega, \mathscr{F}_T)$. Instead, the DM assumes multiple first-order probability measures on $(\Omega, \mathscr{F}_T)$. We define Γ as the set of the mutually equivalent first-order probability measures, $\{\mathbb{Q}^\theta\}_{\theta\in\Theta}$, where Θ is a finite space of parameters, θ. Γ contains the reference probability measure, $\mathbb{P}$. The conditional expectation under $\mathbb{P}$ is denoted by $E_t[\cdot] \equiv E[\cdot|\mathscr{F}_t]$, whereas that under $\mathbb{Q}^\theta$ is denoted by $E_t^\theta[\cdot]$.

Each $\theta \in \Theta$ is a R^k-valued $\mathscr{F}_t$-adapted process, $\theta = \{\theta_t\}_{t=0}^T$, satisfying the Novikov condition:

$$E_0\left[\exp\left\{\frac{1}{2}\int_0^T \|\theta_s\|^2 ds\right\}\right] < \infty.$$

Then, in our Brownian setting, we can construct the first-order probability measure, $\mathbb{Q}^\theta$, corresponding to each θ through the following Radon–Nikodým derivative process with respect to $\mathbb{P}$:

$$\xi_t^\theta \equiv \left.\frac{d\mathbb{Q}^\theta}{d\mathbb{P}}\right|_{\mathscr{F}_t} = \exp\left\{-\frac{1}{2}\int_0^t \|\theta_s\|^2 ds + \int_0^t \theta_s^\top dBs\right\}, \quad \xi_0^\theta = 1.$$

[2] Gollier (2011) indicates that, at the optimum, the DM's behavior under the KMM model is observationally equivalent to that under the SEU model with distorted beliefs.

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