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Shipwrecks and treasure hunters☆

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ABSTRACT

We examine dynamic search as a game in which two rivals explore (an island) for a hidden prize of known value. In every period until its discovery, the players decide how much of the unsearched area to comb. If a player finds the prize alone he wins it and the game ends. Players have a per-period discount factor and costs proportional to the area they search. First, as a benchmark for efficiency, we solve the one-player search problem. Second, in the two-player setting we show that typically there is inefficient over-search – a result akin to the tragedy of the commons. However, for players with intermediate levels of patience, there is the possibility of inefficient under-search as players incorporate the expected future payoffs in their current search decisions. Finally, with patient players, several counterintuitive results can arise: for example, players might be better off searching a larger island or looking for a less valuable prize.

1. Introduction

In 1708 the *San José*, a Spanish galleon, was sunk in a battle with British warships near Barú, off the Colombian coast, carrying gold, silver and gems from the New World. Some three hundred years later, the Colombian government announced they located the shipwreck in December 2015, in a find estimated to be worth $1bn. Even if the potential bounties involved are smaller, the seas are littered with sunken treasure ships, such as; the S.S. Central America, sunk in 1857 along with 15 tons of gold off in the Carolinas; the S.S. Republic which sank with 51,000 U.S. silver and gold coins off the Georgian coast in 1865 (discovered in 2003); and the Whydah Gally, the 18th century galley discovered in 1984 with loot valued at more than $400 million. Often these shipwreck treasure troves have similar characteristics. Firstly, while it might be broadly known where the ship sank – like a specific coastline, sea, or in the vicinity of a particularly treacherous pass – the exact location of shipwreck is unknown. This means, consequently, that there is uncertainty about the total search costs involved in discovering the wreck. Secondly, there is often quite accurate information about the value of the sunken treasure, with historical records detailing how much gold or other valuable items were on board; for example, the *San José* is thought to have been carrying 11 million 8-escudos gold coins. Thirdly, given the potential prize, often several interested rival teams are in a race to make the same find, each with the hope of being the first to discover the bounty and claim the prize.

Beyond looking for sunken shipwrecks laden with treasure, strategic search is ubiquitous: companies look for new profitable products and markets; prospectors search for unexploited deposits; and pharmaceutical companies engage in R&D in

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the hope that they discover the next generation of drugs. We analyze a dynamic model in which two players search for a treasure hidden somewhere in a given area (on an island or in a particular sea). The value of the treasure is common knowledge, and search is costly. Once the treasure is found the game ends. In each period, both players make their search decisions simultaneously. If the treasure is not discovered the game continues to the next period in which the players have to decide how much of the unsearched area to explore. If the two players find the treasure simultaneously (by both searching the correct location), whilst each incurs their own costs, the treasure is destroyed resulting in a payoff of zero for both parties.[1] We consider the case when search is observable; that is, players are informed about the areas that have already been searched by their opponents. For tractability, we also restrict the total search cost to be not more than the value of the treasure.

In this setup, each state can be described by the remaining unsearched area. While there could be multiple subgame perfect equilibria in the game, we show that, except for possibly one point, the symmetric Markov perfect equilibrium (SMPE) is unique.[2] Within this framework, we compare the two-player SMPE to the efficient search outcome (with one player). Relative to a single-player, multi-player search is typically inefficient, except for very small islands, when players behave as a cartel and search lasts just one period.[3] In the case of slightly larger but still small islands, multiple players search too fast, akin to the *standard tragedy of the commons*; players search too much in the current period, because the probability of finding a treasure is relatively high and each player does not consider how their search imposes a spillover cost on their rival. This leads to inefficient over-search. For example, in a race to be the first to discover a sunken treasure ship, each team might be tempted to establish an intensive search program. While this can hasten discovery, it also dissipates potential rents in the process. In the case of large islands, the incentive to search more in the current period is not as strong. In fact, with a large search area the parties can even inefficiently under-search. The intuition for this is that with a large island, the parties shade their search, in the hope to free-ride on their rival's unsuccessful attempts. Moreover, the incentive to search in early periods could be further dampened if both parties anticipate the possibility of a competitive 'search war', which could arise as the unsearched area shrinks (and the island effectively becomes small). Note that this is a new result; in the present model, in contrast to much of the existing literature, both over- and under-search can endogenously arise along the equilibrium path within the same project.

As search is costly, it seems natural to conjecture that a smaller island (which lowers expected total search costs) is better than a bigger island for all players. This is not necessarily the case; in fact, expected payoffs can increase with the potential search area. If the island is of moderate size (not so small that all of the island is searched immediately), the tragedy of the commons effect is strong, and players over-search. If the island area is increased, the tragedy of the commons effect is diminished, and players search the island more efficiently. It turns out that this efficiency improvement may be large enough to outweigh the increase in the cost of searching the larger island. Similar non-monotonic results apply to the other key parameters as well; players can be better off with a smaller prize or with higher search costs. These results have implications for the design of search competitions for new pharmaceutical products or other innovations, and for mineral exploration.[4] The potential non-monotonicities could also explain why search for a shipwreck, for instance, can wax and wane over time.

The focus in this paper is on the dynamics of investment in relation to search for a private good. Previous work on private goods mostly deals with situations that are either static or involve complete information.[5] The typical outcome of these models is that firms overinvest. One of the few papers that considers the dynamics of investment is Reinganum (1981), who shows that in a dynamic R&D race where each of the two firm chooses a time path of expenditures, firms can either underinvest or overinvest (but not both in the same game) as compared with the choice of a single searcher. Aggregate expenditure on R&D may therefore be either too high or too low relative to the efficient outcome depending on the exogenous parameters. In her model, Reinganum assumes that the success function is exponential and the environment is stationary;[6] this means that equilibrium strategies can be represented as functions of time only, simplifying the analysis.[7]

There are many situations, however, where the memorylessness assumption is not satisfactory; it is the welfare analysis of two-player search in a dynamic environment that is a key contribution of the model in this paper. The setup we use here is similar to Matros and Smirnov (2016), who analyze duplication in search.[8] They find that with independent search, all rents are dissipated. Here, our focus is on (in)efficiency, not duplication. We find that, while search is typically not efficient,

[1] This assumption is standard in the R&D literature. Intuitively, if several players discover the treasure simultaneously, fierce competition (or a legal dispute) runs down the surplus to zero.

[2] As well as simplifying our analysis, imposing Markov perfection makes our results directly comparable to those in the previous literature. See Maskin and Tirole (1988), Bhaskar et al. (2012) and Battaglini et al. (2014) for a general discussion of why the use of SMPE is appropriate. The focus on symmetric MPE is supported by the fact that expected combined value in the asymmetric MPE is never more than in the SMPE; see Section 4.

[3] The rivals' search is efficient not only for small island sizes but also when the cost of searching the entire unexplored area is exactly equal to the value of the treasure, and the players have a discount factor that is sufficiently low. We discuss this case in Section 3.3.

[4] Examples of such research contests include the six Millennium Prize Problems, a 1992 refrigerator competition (see Taylor, 1995), a 1829 steam locomotion tournament (see Fullerton and McAfee, 1999), a 1714 British contest for a method of determining longitude at sea (see Che and Gale, 2003)

[5] See, for example, Loury (1979), Dasgupta and Stiglitz (1980a,b), Lee and Wilde (1980), Reinganum (1989) and Long (2010) for a survey of the literature.

[6] In the context of our model, this would be equivalent to assuming that the present value of the treasure is constant.

[7] Reinganum (1982) uses a similar framework to show how the availability of patent protection can accelerate development of the innovation.

[8] Two other papers that focus on duplicative search are Chatterjee and Evans (2004) and Fershtman and Rubinstein (1997). In Chatterjee and Evans (2004), in each period players decide whether to research one or two project, where only one project will eventually be successful. Inefficient duplication