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Comments on "Exploiting MIT shocks in heterogeneous-agent economies: The impulse response as a numerical derivative" by T. Boppart, P. Krusell and K. Mitman[☆]

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1. Introduction

Heterogeneous agent (HA) models are becoming more and more important in economics, the reasons for which are nicely summarized in the paper. The numerical solution of these models is often a challenge, and I fully agree with the authors that it is very desirable to have a variety of computational methods available, each of which is useful for some models and not so useful for others. The method proposed by Boppart, Krusell and Mitman (henceforth BKM) is a very welcome addition to the available set of tools, given its conceptual and computational simplicity, relative to other widely used methods. They demonstrate the usefulness of their method on a relatively standard HA model. Because of space limitations, I will not comment on the specific application, but rather give a discussion of the general solution method.

2. Explaining the method

The method of BKM builds on three themes:

1. Sequence form versus recursive form of a solution.

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2. Linearity
3. Certainty equivalence

Let us start with a discussion of the **sequence vs. recursive form** of a solution. The solution to most stochastic dynamic models can be written in two different forms. The first one is the recursive form

$$d_t = d(S_t) \tag{1}$$

where the vector of equilibrium variables (decisions, prices etc.) d_t is written as a function of the vector of states S_t. The second one is the sequence form, where equilibrium variables are written as a function of the complete history of exogenous shocks z_t:

$$d_t = d(z_t, z_{t-1}, z_{t-2}, \dots) \tag{2}$$

In representative agent DSGE models, S_t is a vector of fixed finite dimension, containing perhaps a few dozen variables. The recursive form (1) is therefore a much more parsimonious representation of the solution than (2). In HA models, S_t includes the whole cross-sectional distribution of individual states, and is usually of infinite dimension. If the model is driven by very few shocks, the sequence form is probably more parsimonious, a fact that BKM exploit. This insight is not new; it was used for example in Chien et al. (2011). But it seems to me that the use of the sequence form is clearly under-researched and under-utilized.

BKM realize that the sequence form is particularly useful if combined with **linearization**. Linearization has been the most widely used method to solve DSGE models for many years now, and it has been adapted to HA models in Reiter (2009): while the individual agents' decision is solved as a nonlinear function of their own individual states, the dependency of this nonlinear decision function on the vector of aggregate states is linearized. This allows to keep track of a high-dimensional representation of this distribution as part of the aggregate state.

BKM exploit the linearity of the sequence form $d(z_t, z_{t-1}, z_{t-2}, \dots)$, which means that

$$d(z_t, z_{t-1}, z_{t-2}, \dots) = z_t d(1,0,0,0,\dots) + z_{t-1} d(0,1,0,0,\dots) + z_{t-2} d(0,0,1,0,\dots) + z_{t-3} d(0,0,0,1,\dots) + \dots \tag{3}$$

Notice that, for example, $d(0,0,0,1,\dots)$ is just the impact today of a unit impulse three periods ago. Eq. (3) therefore writes d as a superposition of impulse response functions, which shows that we can recover the linear solution in sequence form from one impulse response function. This is the key element of the BKM algorithm.

The only difficult task is to compute this impulse response function. BKM do this not through linearization; they rather compute a fully nonlinear impulse response function based on **certainty equivalence**. The certainty equivalence principle is a well known theorem from dynamic optimization, saying that the variance of the exogenous shocks has no effect on the optimal policy function (policy as a function of the state) if the objective function is quadratic, transition functions are linear, and the shocks enter additively. These assumptions are almost never satisfied exactly in economic models, but they are often a good enough approximation. In particular, linearization of the first order conditions and other equilibrium conditions leads to decision functions that are linear in the state vector and do not depend on the variance of shocks.[1] Assuming certainty equivalence is therefore no disadvantage relative to other linearization techniques. BKM use what I would call a "certainty equivalence impulse response function" (CEIRF): the deterministic foresight path that obtains in the model if the exogenous shock starts away from its steady state value, and assumes its conditional expectation in all future periods with probability one.

Combining the three themes, BKM compute the model solution in sequence form by a linear superposition of CEIRFs:

Algorithm BKM:

1. Compute a CEIRF x_k for $k = 0, \dots, K$ with an initial shock of size $\bar{z}$; in the paper, they set $\bar{z}$ to one standard deviation of the shock.
   Define the scaled impulse response $\hat{x}_k = x_k/\bar{z}$ as the impulse per unit shock.
2. Write the model solution X_t as a moving average of past shocks:

$$X_t = \sum_{k=0}^{K} \hat{x}_k z_{t-k} \tag{4}$$

The computation of perfect foresight transition paths has been used in large scale OLG models at least since Auerbach and Kotlikoff (1981). The novelty of algorithm BKM lies is step 2, the superposition of CEIRFs to obtain the stochastic properties of the model solution.

Because the CEIRF is calculated by nonlinear methods, it gives rise to a natural linearity test. If linearity is a good approximation, the choice of $\bar{z}$ should have little effect on the scaled impulse response function. One can vary $\bar{z}$ within a reasonable range (say ± 5 standard deviations of the shock) and see whether this is true. Using this test, the authors find that linearity is quite well satisfied in their model.[2]

[1] Notice that in nonlinear models, CE holds asymptotically for $\sigma \to 0$, because the effect of uncertainty is $O(\sigma^2)$, cf. Schmitt-Grohé and Uribe (2004).

[2] This test was already used in Reiter et al. (2013, Section 7.2).