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Invariance of the equilibrium set of games with an endogenous sharing rule ☆

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Abstract

We consider games with an endogenous sharing rule and provide conditions for the invariance of the equilibrium set, i.e., for the existence of a common equilibrium set for the games defined by each possible sharing rule. Applications of our results include Bertrand competition with convex costs, electoral competition, and contests.

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1. Introduction

Consider the classical problem of a Bertrand duopoly, i.e., two firms set prices to compete for customers. A difficulty in modeling this situation is that when the firms set the same price, customers are indifferent with respect to where to buy, so that it is unclear how to specify the

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firms' profits (payoffs). Games with an endogenous sharing rule, introduced by Simon and Zame (1990), avoid this difficulty by specifying players' payoffs by a correspondence rather than by a function, thus taking a broad stand on how to specify players' payoffs.

When analyzing a game with an endogenous sharing rule, one may be interested in obtaining the existence of a strategy profile that is an equilibrium for each possible sharing rule, henceforth, an invariant equilibrium. Indeed, whenever such invariant equilibrium exists, the choice of the sharing rule becomes immaterial. Specifically, the predictions provided by such invariant equilibrium are robust to the actual sharing rule that happens to occur, e.g. the actual choice made by consumers regarding which of the two duopolists to buy from.

An appealing scenario occurs when each strategy that is an equilibrium for some sharing rule is an invariant equilibrium, henceforth, when the equilibrium set is invariant. In this case, any equilibrium is robust to the actual sharing rule that happens to occur and, based on this robustness notion, there is no need to select amongst the set of equilibria. Furthermore, as pointed out by Lebrun (1996) and Jackson and Swinkels (2005), the invariance of the equilibrium set is also important from a practical viewpoint. Indeed, it allows us to analyze the equilibrium set of the game defined by a sharing rule we may be interested in by analyzing the equilibrium set of the game defined by any other (simpler, easier to analyze) sharing rule. In particular, in the presence of incomplete information, it is often easier to establish the existence of equilibrium for some type-independent sharing rule by first showing that some type-dependent sharing rule has an equilibrium and then appeal to the invariance of the equilibrium set.

In this paper, we establish results concerning the invariance of the equilibrium set for general games with an endogenous sharing rule. Our first key condition, called "virtual continuity," roughly says that each player can, with a probability close to one, avoid points at which the payoff correspondence is multi-valued while virtually achieving the same payoff given the strategies of the other players, regardless of the particular sharing rule which is in force.[1] We show that, under this condition, the equilibrium set coincides with the set of invariant equilibrium in the class of games defined by efficient sharing rules. More precisely, any strategy that is an equilibrium of the game defined by some sharing rule is also an equilibrium in the game defined by any efficient sharing rule. This means that for equilibrium analysis of virtually continuous games with an endogenous sharing rule, one may focus on efficient sharing rules. In this light, our result has the interesting interpretation that, in equilibrium, indeterminacies are resolved efficiently. Moreover, as we illustrate using simple Bertrand examples, this result is also useful to compute the equilibrium set of games with an endogenous sharing rule.

Our second key condition, called "strong indeterminacy," roughly requires that indeterminacies are not eliminated by focusing on efficient sharing rules. More precisely, if a player has, at some action profile, more than one possible payoff, then there are at least two efficient payoff profiles at that action profile giving different payoffs to that player. Our main result states that each game with an endogenous sharing rule satisfying virtual continuity and strong indeterminacy has an invariant equilibrium set.

The intuition for our results is as follows. First, virtual continuity implies that each player's value function (i.e. the function assigning to each strategy profile of the other players the supremum of the payoffs he can achieve) is the same for all sharing rules. Second, any equilibrium for some sharing rule remains an equilibrium for any efficient sharing rule. This is so because

[1] Virtual continuity generalizes analogous conditions that appeared in Dasgupta and Maskin (1986a, Theorem 5), Jackson and Swinkels (2005, Lemma 7) and Bagh (2010, Theorem 4.2).