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Sharing sequential values in a network ☆

Ruben Juarez [a,*], Chiu Yu Ko [b], Jingyi Xue [c]

[a] *University of Hawaii, United States of America*
[b] *National University of Singapore, Singapore*
[c] *Singapore Management University, Singapore*

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Abstract

Consider a sequential process where agents have individual values at every possible step. A planner is in charge of selecting steps and distributing the accumulated aggregate values among a number of agents. We model this process by a directed network, whereby each edge is associated with a vector of individual values. This model applies to several new and existing problems, e.g. developing a connected public facility and distributing total values received by surrounding districts, selecting a long-term production project and sharing final profits among partners of a firm, or choosing a machine schedule to serve different tasks and distributing total benefits among task owners.

Herein, we provide the first axiomatic study on path selection and value-sharing in networks. We consider four sets of axioms from different perspectives, including those related to (1) the sequential consistency of value-sharing; (2) the monotonicity of value-sharing with respect to technology improvements; (3) the independence of value-sharing with respect to certain network transformations; and (4) the robust implementation of the efficient path selection when the planner has no information about network configuration. Surprisingly, these four disparate sets of axioms characterize similar classes of solutions, namely selecting an efficient path(s) and assigning to each agent a share of total values that is independent of individual values. Furthermore, we characterize more general solutions that depend on individual values.

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* Corresponding author.
E-mail addresses: rubenj@hawaii.edu (R. Juarez), kochiuyu@nus.edu.sg (C.Y. Ko), jyxue@smu.edu.sg (J. Xue).

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1. Introduction

The axiomatic division of costs and benefits has been studied extensively over the past 60 years, starting with bargaining (Nash, 1950) and cooperative games (Shapley, 1953), and followed by applications to problems such as rationing and bankruptcy (O'Neill, 1982; Aumann and Maschler, 1985; Thomson, 2003; Moulin, 2000, 2002), airport cost-sharing (Littlechild and Owen, 1973; Thomson, 2013), hierarchical and group ventures (Hougaard et al., 2017; Juarez et al., 2018), and more general cost-sharing problems (e.g., Sprumont, 1982, 1998; Moulin, 1994; Friedman and Moulin, 1999; Moulin and Sprumont, 2005; Moulin and Shenker, 1992; Juarez, 2013, 2008). Such studies have characterized a wide variety of sharing rules using axioms motivated by positive and normative perspectives. However, they are largely limited to scenarios with a fixed resource and little is known regarding scenarios that are more general in two respects: (1) the amount of the resource may not be fixed but can be chosen, and (2) the resource may be generated in a sequence of steps, where the amount in future steps depends on the choices made in previous steps. Such a dynamic problem requires resource-generating steps to be determined together with the allocation. This "two-tiered" approach not only expands the range of problems, but also gives rise to a new question on the interdependence of the step selection and sharing rules.

To illustrate our problem better, consider a planner in charge of developing a connected public facility (e.g. a highway, railroads, or an irrigation canal). The project might be developed in different steps, each of which might produce different benefits to the agents in a given society. The planner is in charge of choosing the steps and redistributing the benefits of the project among the agents. After proceeding along each step, the planner faces a new problem which is different from the original one and might depend on the preceding steps (Section 1.1 discusses other applications).

Formally, a finite number of agents are facing a sequential process generating individual values that can be redistributed among themselves. A sequential process, or simply **process**, consists of a network and a value function associated with the network. A network is an acyclic and connected finite directed graph with a unique source and possibly multiple sinks. Each node in the network represents a stage at which (1) a step to continue should be chosen (except for a sink) and (2) individual values generated until then could be redistributed. Each edge in the network represents a feasible step to continue. A value function associated with the network assigns to each edge a value vector that specifies for each agent his individual value generated at the step.

Given a process, a planner is in charge of (1) selecting a way of proceeding from the source to a sink which is called a **path**, and (2) redistributing the sum of individual values accumulated over all edges along the path which is called **the value of the path**. A **solution** consists of a path selection rule and a sharing rule which respectively recommends for each process a set of paths with the same value and an allocation of the value among agents.

We provide the first systematic and comprehensive study of this problem by considering axioms appropriate to a wide range of scenarios. Surprisingly, our four sets of axioms from different perspectives characterize similar classes of solutions — selecting the path(s) with the highest value (hereafter referred to as the efficient path(s)) and assigning to each agent a share of the value