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The single-peaked domain revisited: A simple global characterization ☆

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Abstract

It is proved that, among all restricted preference domains that guarantee consistency (i.e. transitivity) of pairwise majority voting, the single-peaked domain is the only minimally rich and connected domain that contains two completely reversed strict preference orders. It is argued that this result explains the predominant role of single-peakedness as a domain restriction in models of political economy and elsewhere. The main result has a number of corollaries, among them a dual characterization of the single-dipped domain; it also implies that a single-crossing ('order-restricted') domain can be minimally rich only if it is a subdomain of a single-peaked domain. The conclusions are robust as the results apply both to domains of strict and of weak preference orders, respectively.

JEL classification: D71; C72

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1. Introduction

A subset of preference orders on a finite set of alternatives is called *single-peaked* if there exists a left-to-right arrangement of alternatives such that all upper contour sets are connected ('convex') with respect to the given left-to-right arrangement of alternatives. The celebrated median voter theorem of Black (1948) and Arrow (1951) states that the domain of all single-peaked linear orders with respect to a fixed underlying spectrum of alternatives form a 'Condorcet domain,' i.e. pairwise majority voting with an odd number of individuals each of whom has preferences from the given domain induces a transitive relation. Moreover, the domain of all single-peaked preferences is *minimally rich* in the sense that every alternative is on top of at least one preference order; it is *connected* in the sense that every two single-peaked orders can be obtained from each other by a sequence of transpositions of neighboring alternatives such that the resulting order remains single-peaked at each step; and it contains two *completely reversed* orders (namely, the orders that rank the alternatives in descending order from left to right and from right to left, respectively).

This paper's main result (Theorem 1) shows that, conversely, every minimally rich and connected Condorcet domain which contains at least one pair of completely reversed orders must be single-peaked.[1] As is easily verified, any single-peaked domain contains at most *one* pair of completely reversed orders. We thus obtain as a corollary that, for any given pair of completely reversed orders, there is a unique *maximal* Condorcet domain that contains them and is minimally rich as well as connected: the domain of *all* orders that are single-peaked with respect to either one of the given pair of completely reversed orders (Corollary 1).

This result is remarkable in particular in view of the fact that quite a number of non-single-peaked Condorcet domains have been identified in the literature, among others the domains satisfying Sen's 'value restriction' condition (Sen, 1966) with the 'single-dipped' domain (Inada, 1964) as a special case, the domains satisfying the so-called 'intermediateness' property (Grandmont, 1978; Demange, 2012), and the 'order-restricted' domains identified by Rothstein (1990); the latter domains are sometimes also referred to as the domains with the *single-crossing property* (Gans and Smart, 1996; Saporiti, 2009; Puppe and Slinko, 2017). Our analysis shows that none of these domains can jointly satisfy the three conditions of minimal richness, connectedness and the inclusion of a pair of completely reversed orders unless it is *also* single-peaked. In particular, a single-crossing domain can be minimally rich only if it is at the same time single-peaked (Corollary 3).

The purpose of the present analysis is not to justify the assumption of single-peakedness *per se* and, in fact, the empirical evidence for single-peakedness is mixed, see the review of the literature below. The main argument put forward here is that, among all domains that guarantee consistency of pairwise majority voting, the single-peaked domain is distinguished by a remarkably simple set of additional requirements: connectedness, minimal richness and the existence of two completely reversed orders. The main conclusion to be drawn from the present analysis is

[1] In fact, as detailed in Section 2 below, the condition of connectedness can be substantially relaxed in this result to the condition that there exist *one* path that connects a pair of completely reversed orders.