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Finding a stable matching under type-specific minimum quotas

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Abstract

In matching problems with minimum and maximum type-specific quotas, there may not exist a stable (i.e., fair and non-wasteful) assignment (Ehlers et al., 2014). This paper investigates the structure of schools' priority rankings which guarantees stability. First, we show that there always exists a fair and non-wasteful assignment if for each type of students, schools have common priority rankings over a certain number of bottom students. Next, we show that the pairwise version of this condition characterizes the maximal domain of two schools' priority rankings over same type students to guarantee the existence of stable assignments. To prove the existence theorem, we propose a new mechanism *Deferred Acceptance with Precedence Lists* (DAPL), which is feasible, non-wasteful, strictly PL-fair and group strategy-proof for any priority rankings. Strict PL-fairness is weaker than fairness, but DAPL satisfies fairness under our sufficient condition. We also show that there is no strategy-proof mechanism that Pareto dominates DAPL whenever the outcome of DAPL is Pareto dominated by a stable assignment.

JEL classification: C78; D47; D82

Keywords: Type-specific minimum quotas; Stability; Priority rankings; Deferred acceptance; Controlled school choice

1. Introduction

While the modern matching theory has provided solutions to many real-world allocation problems, there are still important problems that have yet to be solved due to technical difficulties.

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One of such problems is matching with *type-specific minimum quotas*. In many school districts in the United States, maximum and minimum quotas are separately set for different groups of students in order to achieve gender, racial or socioeconomic diversity at schools. For instance, New York City requires Educational Option (EdOpt) schools to keep the diversity of ability levels by accepting students from different ranges of test scores. For each EdOpt school, 16 percent of students must score below the grade level on the standardized test, 68 percent must score at the grade level, and 16 percent must score above the grade level (Abdulkadiroğlu et al., 2005). In the public school choice program in Cambridge, US, students are classified into high and low socioeconomic groups, and each school is required to admit certain percentages of students from each group (Fragiadakis and Troyan, 2017). Type-specific minimum quotas are also observed in the matching problems between students and supervisors (Kawagoe and Matsubae, 2017). University departments (especially in natural science) set minimum quotas to supervisors because (i) supervisors need some students to operate their laboratories, and (ii) the departments aim to achieve an equal share of educational burden among the supervisors. And in many cases, these constraints only apply to students from a certain program or field, which requires quotas be type-specific.

Despite the prevalence of type-specific minimum quotas, theoretically proposing a desirable solution to this problem is still a difficult task. Ehlers (2010) and Ehlers et al. (2014) formulated this problem and found a general impossibility result that the set of feasible, fair and non-wasteful assignments may be empty (Theorem 1). This is in contrast with the standard matching problem with only maximum type-specific quotas, where we can always find a stable assignment by the Deferred Acceptance (DA) mechanism (Abdulkadiroğlu and Sönmez, 2003; Abdulkadiroğlu, 2005). And in most of the literature which pursued stability in this problem, the authors proposed solutions by (i) interpreting constraints as soft bounds (Ehlers et al., 2014) or (ii) dropping or weakening one of (or both of) fairness and non-wastefulness (Ehlers et al., 2014; Fragiadakis and Troyan, 2017; Goto et al., 2016, 2017).

In this paper, we propose designing the priority rankings of schools to ensure stability without weakening stability itself.[1,2] As we discuss in subsection 3.1 and Section 4, the priority structure is part of the choice variables for the mechanism designer in many applications. To design priority rankings appropriately, we need to answer the following question: What is the domain of priority rankings for which we can (or cannot) ensure the existence of fair and non-wasteful assignments under type-specific constraints? We first provide a positive answer to this. When schools have common priority among a certain number of bottom students for each type (we call this condition *B-common priority*), we show that there always exists a feasible, fair and non-wasteful assignment (Theorem 2). The threshold of bottom students varies across schools, and it is computed by the capacity and type-specific constraints of all schools. Intuitively, if a student is ranked lower than other same type students at some school (and if her ranking is low enough), then she is so in any other school as well. The priority order between different types of students can differ across schools in an arbitrary way.

Second, we also show that the pairwise version of B-common priority characterizes the maximal domain of two schools' priority rankings over same type students to guarantee the existence

[1] We employ the most natural definition of stability proposed by Ehlers et al. (2014) in our model.

[2] In the literature, the domain of schools' priority rankings is studied to understand the relationship among several properties such as efficiency, stability and strategy-proofness for school choice mechanisms (Ergin, 2002; Kesten, 2006; Haeringer and Klijn, 2009; Kumano, 2013).