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# Dynamic directed random matching ☆

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## Abstract

We develop a general and unified model in which a continuum of agents conduct directed random searches for counterparties. Our results provide the first probabilistic foundation for static and dynamic models of directed search (including the matching-function approach) that are common in search-based models of financial markets, monetary theory, and labor economics. The agents' types are shown to be independent discrete-time Markov processes that incorporate the effects of random mutation, random matching with match-induced type changes, and with the potential for enduring partnerships that may have randomly timed break-ups. The multi-period cross-sectional distribution of types is shown to be deterministic and is calculated using the exact law of large numbers.


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## 1. Introduction

The economics literature is replete with models that assume independent random matching among a continuum of agents.[1] The agents in these models are frequently motivated to conduct "directed search," that is, to focus their searches toward those types of counterparties that offer greater gains from interaction, or toward those types that are less costly to find. For example, Rogerson et al. (2005) describe cases in which "search is directed – i.e., workers do not encounter firms completely at random but try to locate those posting attractive terms of trade." Our central marginal contribution is to provide a mathematical foundation for the existence and properties of directed search models.

Independent directed random matching, which includes the popular "matching-function" approach, is the key to achieving tractability in many search-based models of financial markets, monetary theory, and labor economics.

Previous work on mathematical foundations for random matching considers only search that is "undirected," in the sense that, conditional on a match by a given agent at a given time, the probability that the match is with a particular "target" type of agents is merely the fraction of agents of the target type. Directed search can arise, for example, when one side of a market posts terms of trade that are especially attractive to specific types of agents.

Despite heavy reliance in the economics literature on models of independent directed search,[2] until now there has actually been no demonstration of the existence of such search models, nor of the assumed aggregate behavior of these models that is supposedly based on the law of large numbers. This paper demonstrates the existence and properties of general models of static and dynamic independent directed search, thus placing a complete mathematical foundation under the directed-search models assumed in the prior literature. Our results include new features and properties that may be useful in future research.

Earlier foundational work on random matching in a dynamic setting, which we review in Section 5, also presumes that partnerships break up immediately after matching. Here, we allow for the potential of enduring partnerships, which may have randomly timed break-ups. In order to meet the objectives of this paper, a completely new methodology is required, for both static and dynamic settings.[3]

We first consider a static setting in which search is "directed," in the sense that the probability $q_{kl}$ that an agent of type $k$ is matched to an agent of type $l$ can vary with the respective types $k$ and $l$, from some type space $S$. We first show, in Theorem 1, the existence of directed random

1 Hellwig (1976) is the first, to our knowledge, to have relied on the effect of the exact law of large numbers for random pairwise matching in a market. Other examples include Binmore and Samuelson (1999), Currarini et al. (2009), Duffie et al. (2005), Green and Zhou (2002), Kiyotaki and Wright (1989), Lagos and Rocheteau (2009), Vayanos and Weill (2008), and Weill (2007).

2 Among the many applications of directed search in the economics literature, in addition to those cited elsewhere in this paper, are the models of Daron and Shimer (1999), Albrecht et al. (2006), Burdett et al. (2001), Camera and Selcuk (2009), Eeckhout and Kircher (2010), Faig and Jerez (2005), Guerrieri et al. (2010), Kiyotaki and Lagos (2007), Li et al. (2012) McAfee (1993), Menzio (2007), Moen (1997), Peters (1991), Shi (2002), and Watanabe (2010).

3 See the discussions in the first two paragraphs of Subsection E.1 on the proof of the static results, and the second paragraph of Subsection E.2 on the proof of the dynamic results, respectively.