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Notes

Uniqueness and stability of equilibrium in economies with two goods ☆

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Abstract

We offer new sufficient conditions ensuring demand is downward sloping local to equilibrium. It follows that equilibrium is unique and stable in the sense that rising supply implies falling prices. In our setting, there are two goods, which we interpret as consumption in different time periods, and many impatience types. Agents have the same Bernoulli utility function, but the types differ arbitrarily in time preference. Our main result is that if endowments are identical and utility displays nonincreasing absolute risk aversion, then market demand is strictly downward sloping local to equilibrium. We discuss implications for the literature surrounding Diamond and Dybvig (1983).

JEL classification: C62; D51; D58

Keywords: Uniqueness of equilibrium; Absolute risk aversion; Excess demand functions; Stability of equilibrium; Diamond–Dybvig models

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1. Introduction

When the aggregate endowment temporarily increases, will interest rates fall? Will discoveries of oil push down gas prices? The intuitive answer to these questions is yes. The logic is that since individual demand is normally downward sloping, if supply of a good increases, its price must fall to clear markets and maintain equilibrium. But this reasoning requires that aggregate, market demand inherit the properties of individual demand. If, for example, markets are complete and agents have identical homothetic utility functions, equilibrium prices are as if there were a representative agent with homothetic preferences. In this case, micro intuition extends to the macroeconomy. But what if we place less restrictive assumptions on individual preferences? Will market demand still look like the demand curve of a rational person?

The Sonnenschein (1972, 1973)–Mantel (1974)–Debreu (1974) (SMD) results give a negative answer to this question (see Shafer and Sonnenschein, 1982 for a survey). They say that arbitrary continuous market excess demand functions can be generated by individuals with positive endowments and continuous, increasing, concave utility functions. In the case of Mantel (1976), the utility functions can even be restricted to be homothetic. The striking implication is that strong assumptions about individuals (such as homotheticity) may yield wild market excess demand functions exhibiting, for example, multiple equilibria and thus equilibria with upward sloping demand. In this case, equilibrium may be unstable in the sense that increasing supply may lead to *higher* prices. In short, the SMD results show that concavity, continuity, and homotheticity are not sufficient for aggregate demand to behave like individual demand. See Toda and Walsh (2017) for examples of and sufficient conditions for unstable equilibria in Edgeworth box economies with identical homothetic or quadratic Bernoulli utility functions.

Yet theorists have uncovered many cases where competitive equilibrium is unique and thus stable, meaning that aggregate demand is downward sloping at least local to equilibrium. See Kehoe (1998) and Mas-Colell (1991) for surveys of the uniqueness literature.[1] For example, suppose agent $i \in I$ has differentiable, increasing, concave utility $u_i(x) = \sum_{j=1}^{J} u_{i,j}\left(x_j\right)$ over J goods and a positive endowment of each good. If for all $i \in I$ relative risk aversion is everywhere less than 1, $-x_j u''_{i,j}\left(x_j\right)/u'_{i,j}\left(x_j\right) < 1$, then all excess demands functions are downward sloping, and the resulting equilibrium is unique and stable (see Mas-Colell et al., 1995). By assuming collinear endowments, the result of Mitiushin and Polterovich (1978) weakens this condition to $-xu''/u' < 4$. Chipman (1974) and Eisenberg (1961) show that with collinear endowments and homothetic utilities, aggregation (which implies uniqueness) is possible with any risk aversion. However, as Kehoe (1998) observes, "useful conditions that guarantee uniqueness of equilibrium are very restrictive," involving, say, *quantitative* bounds on relative risk aversion.[2] Generally, as Kehoe (1998) continues, conditions sufficient for uniqueness have been difficult to translate into economic intuition without losing necessity. Furthermore, while there are many applied general equilibrium models for which we do not have uniqueness proofs (as in the infinite horizon macroeconomics literature), non-uniqueness examples are equally rare in some settings. Therefore, as Kehoe (1998) writes, "It may be the case that most applied models have unique

[1] See Negishi (1962), Arrow and Hurwicz (1958), and Walras (1954) for early treatments of the topic of stability of competitive equilibrium.

[2] Also, 4 is not a large value for relative risk aversion in the sense that many theoretical and empirical studies assume or estimate relative risk aversion to be well in excess of 4. See, for example, the meta-analysis of Havranek et al. (2015). Note that while their study is about the elasticity of intertemporal substitution (EIS), most of the papers they reference restrict risk aversion to be the reciprocal of the EIS.