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Convex dynamic programming with (bounded) recursive utility ☆

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Abstract

We consider convex dynamic programs with general (bounded) recursive utilities. The Contraction Mapping Theorem fails when the utility aggregator does not obey any discounting property. This failure occurs even with traditional aggregators and certainty equivalent specifications. However, the Bellman operator admits a unique fixed point when an interior policy is feasible. This happens because utility values are unique at interior consumption plans and, when an interior perturbation is feasible, drops in utility values can be avoided.

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1. Introduction

In spite of a growing interest for recursive utility (Koopmans, 1960) in macroeconomics and finance (see, for instance, Backus et al., 2005, Miao, 2014 and Skiadas, 2009), concomitant progress in dynamic programming methods has not occurred in recent years. In this paper, we develop a suitable approach to convex programs for bounded recursive utilities. Acknowledging the failure of the canonical Contraction Mapping Theorem, our technique rests upon the theory of monotone concave operators (Krasnosel'skiĭ, 1964). Under certain interiority restrictions, we prove existence of a unique fixed point to the implied Bellman operator.

The traditional theory for additively time-separable utility is grounded on Blackwell discounting condition (see, for instance, Stokey et al., 1989, Theorem 3.3, Acemoglu, 2009, Theorem 6.9 and Stachurski, 2009, Theorem 6.3.5). This guarantees that the Bellman operator is a contraction and so provides an efficient procedure for the computation of the value function as unique fixed point. The prevailing theory for recursive utility reproduces the logic of this approach (see, for instance, Lucas and Stokey, 1984, Becker and Boyd, 1997 and Miao, 2014). Though the aggregator is not linear in continuation utility, as in the additive case, it is assumed that a suitable form of Blackwell discounting is still satisfied. The drawback of this is that commonly used aggregators do not obey this condition (and even fail any Lipschitz continuity) and, so, do not fall under the domain of Blackwell discounting. Furthermore, and independently of time preference, when some form of non-expected utility is introduced (as in Kreps and Porteus, 1978 and Epstein and Zin, 1989), certainty equivalent might not satisfy the additivity property required to establish Blackwell discounting, rendering the Contraction Mapping Theorem unavailable even when utility is otherwise time additive.[1]

When the Contraction Mapping Theorem is inapplicable, existence and uniqueness of the fixed point to the Bellman operator have to be established alternatively. Under monotonicity, existence might be proved by appealing to Tarski Theorem. Uniqueness remains unverified and, as examples illustrate, might even fail in some circumstances. This is not a merely speculative concern. Indeed, implications for optimal policy might differ dramatically at distinct fixed points, thus causing ambiguity on positive and normative grounds, as well as complications for comparative statics. Furthermore, when the Bellman operator admits multiple fixed points, these might correspond to multiple utility functions all consistent with the given aggregator, so further contributing to the ambiguity in terms of welfare.

We consider monotone concave (Thompson) aggregators for recursive utility, introduced by Marinacci and Montrucchio (2010).[2] One justification for this is that it is the natural hypothesis that ensures monotone concavity of the induced utility function. For convex recursive programs, we construct the canonical Bellman operator. Rather than approaching uniqueness directly, as when one applies (some form of) the Contraction Mapping Theorem, we decompose the argument into two parts. Monotonicity of the Bellman operator is exploited to establish existence of fixed points by convergence of monotone extreme orbits. When lower and upper limits coincide, this yields uniqueness and, on the side, an efficient computational tool. Otherwise, we relocate

[1] An alternative approach is due to Streufert (1990) (see also Osaki and Streufert, 1996 and, recently, Bich et al., 2017) and is based on the bicovergence property, a limiting condition ensuring that the substitution of consumption levels with the very best or worst outcome has no effect in the remote future. This approach is not, however, suitable for our purposes since we deal with general (bounded) aggregators for which bicovergence typically fails.

[2] In a recent paper, Marinacci and Montrucchio (2017) have developed a general Tarski-type approach to monotone concave operators.