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Journal of Economic Theory 173 (2018) 181–200

JOURNAL OF
Economic
Theory

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Stochastic optimal growth model with risk sensitive preferences

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Received 24 September 2015; final version received 21 July 2017; accepted 7 November 2017
Available online 9 November 2017

Abstract

This paper studies a one-sector optimal growth model with i.i.d. productivity shocks that are allowed to be unbounded. The utility function is assumed to be non-negative and unbounded from above. The novel feature in our framework is that the agent has risk sensitive preferences in the sense of Hansen and Sargent (1995). Under mild assumptions imposed on the productivity and utility functions we prove that the maximal discounted non-expected utility in the infinite time horizon satisfies the optimality equation and the agent possesses a stationary optimal policy. A new point used in our analysis is an inequality for so-called associated random variables. We also establish the Euler equation that incorporates the solution to the optimality equation.

JEL classification: C61; C62; D81

Keywords: Stochastic growth model; Entropic risk measure; Unbounded utility; Unbounded shocks

1. Introduction

This paper deals with one-sector stochastic optimal growth models with possibly unbounded shocks and non-negative utilities that are allowed to be unbounded from above. Unbounded

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https://doi.org/10.1016/j.jet.2017.11.005

returns are very common in economic models, see Alvarez and Stokey (1998); Boyd (1990); Durán (2000); Le Van and Morhaim (2002) for the deterministic problems, and Balbus et al. (2014); Durán (2003); Jaśkiewicz and Nowak (2011b); Kamihigashi (2007); Ozaki and Streufert (1996) for stochastic problems. Most of the aforementioned works apply the weighted supremum norm approach introduced by Wessels (1977).[1]

The novelty in our model relies on the fact that the agent has risk sensitive preferences of the form

$$V_t = u(a_t) - \frac{\beta}{\gamma} \ln E_t\left[e^{-\gamma V_{t+1}}\right], \tag{1}$$

where $\gamma > 0$ is a risk sensitive coefficient, $\beta \in [0, 1)$ is a time discount factor, a_t is consumption at time t, u is a felicity function and V_t is the lifetime utility from period t onward. Here, E_t stands for the expectation operator with respect to period t information. The parameter γ affects consumer's attitude towards risk in future utility. The form of preferences in (1) was examined by Hansen and Sargent (1995), who used them to deal with a linear quadratic Gaussian control model and by Weil (1993), who found it appealing in a study of precautionary savings and the permanent income hypothesis. More recently, Backus et al. (2015) studied implications of risk and ambiguity on business cycle fluctuations. In particular, they argued that risk sensitive preferences, of the kind studied in our paper, help to identify the sources of aggregate fluctuations and dynamics of asset prices. In addition, Tallarini (2000) showed that increasing risk aversion significantly improves the model's asset market predictions. As argued by Hansen and Sargent (1995) and Bidder and Smith (2013), risk sensitive preferences are also attractive, because they can be used to model preferences for robustness. In such a case, one can interpret γ in (1) as the robustness parameter. A larger degree of risk aversion is identical to a larger degree of concerns for robustness. For instance, Bidder and Smith (2012) used preferences for robustness to identify animal spirits type of behaviour in the business cycle models. Furthermore, risk sensitive preferences of form (1) have found interesting applications in the problems of Pareto optimal allocations (see Anderson (2005)) or small noise expansions (see Anderson et al. (2012)). The preferences defined in (1) are not time-additive in future utility. Time-additivity, however, requires an agent to be risk neutral in future utility. Risk sensitive preferences, on the other hand, allow the agent to be risk averse in future utility in addition to being risk averse in future consumption.

Our main results are two-fold. First, we establish the optimality equation for the non-expected utility in the infinite time horizon, when the agent has risk sensitive preferences of form (1). The proof as in the standard expected utility case is based on the Banach contraction principle, see Stokey et al. (1989). However, in order to show that the dynamic programming operator maps a space of certain functions into itself, we have to confine our consideration to concave, non-decreasing and non-negative functions that are bounded in the weighted supremum norm. A novel feature in this analysis is an application of some inequality for so-called associated random variables (see Lemma 3). This inequality also plays a crucial role in proving that the fixed point of the dynamic programming operator is indeed the value function. Second, we establish the Euler equation assuming that the production and utility functions are continuously

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[1] The other group of papers makes use of the idea presented by Rincón-Zapatero and Rodriguez-Palmero (2003) within the deterministic framework. Their method rests upon a local contraction and utilises one-sided majorant functions. The extensions of these results to stochastic dynamic programming are reported in Jaśkiewicz and Nowak (2011a); Martins-da Rocha and Vailakis (2010); Matkowski and Nowak (2011). Finally, Ozaki and Streufert (1996) present a different approach based on a formulation of sufficient conditions imposed on the resulted recursive utility function.