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Financial market structures revealed by pricing rules: Efficient complete markets are prevalent ☆

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Abstract

It is well known that when an arbitrage-free financial market is incomplete or has tradable financial assets with frictions there must be multiple risk-neutral probability measures. The main motivation for the present study is to elucidate what type of market structure usually emerges from pricing rules. First, we obtain that finitely generated pricing rules, characterized by polytopes of probabilities, capture the class of all finite arbitrage-free financial markets that are potentially incomplete or subject to frictions affecting tradable assets. Next, we provide a novel characterization of efficient securities and introduce related notions of market completeness that underlies pricing rules. Our main result shows that the class of efficient complete markets with bid-ask spreads is the prevalent case revealed by finitely generated pricing rules.

JEL classification: D52; D53

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1. Introduction

Much of academic research and empirical analysis on valuation of financial assets has been done under the assumption of competitive and frictionless complete markets, *i.e.*, all agents act as price takers and can buy and sell all financial contracts over a given state space without paying any transaction cost. In such context, under the no-arbitrage assumption, securities admit a perfect replication and the well-known linear pricing rule representation holds: the price of any security is given by its (discounted) expected value with respect to a unique risk-neutral probability. However, seminal contributions provide by Hansen and Jagannathan (1991), Heaton and Lucas (1996), and Luttmer (1996) show that the empirical implications of no-arbitrage models are strongly affected by the presence of incompleteness and trading frictions like bid-ask spreads. Jouini and Kallal (1995) show that the incorporation of trade imperfections in competitive securities markets generates pricing rules that are nonlinear but still compatible with the no-arbitrage assumption. In fact, in both cases of trade imperfections, namely incompleteness or frictions affecting tradable securities, the no-arbitrage assumption is equivalent to the existence of multiple risk-neutral probabilities.[1] Furthermore, the multiple linear price characterization of super-replication prices gives that any security has its price computed by the "largest" expected value based on the convex set of risk-neutral probabilities.

In another branch of the literature on asset pricing, the axiomatic approach provide by Chateauneuf et al. (1996), Jouini (2000), Castagnoli et al. (2002), among others,[2] find that general pricing rules are represented by a maximum of expected values over a nonempty, closed and convex set of probability measures (Theorem 4). Thus, the super-replication price functional derived from a particular arbitrage-free financial market can be viewed as a pricing rule represented by a maximum of expected values over the closure of the set of risk-neutral probabilities.

Our main motivation for the present analysis is to study what type of two-period market structure with a finite state space S usually emerges from an arbitrary set of probabilities characterizing a pricing rule $C : \mathbb{R}^S \to \mathbb{R}$. First, we notice that pricing rules characterized by polytopes of probabilities (sets given by convex hulls of a finite set of probabilities), called *finitely generated pricing rules*, capture the universe of all finite arbitrage-free financial markets that are potentially subject to trade imperfections.[3] To the best of our knowledge, all the previous axiomatic contributions on the representation of pricing rules have obtained their functional forms through a general nonempty, closed and convex set of probabilities. However, taking into account the perspective of empirical works where the set of attainable payoffs in a financial economy is generated by a finite set of basic securities (*e.g.*, Subsection 3.3. in Luttmer, 1996), the corresponding set of risk-neutral probabilities has its closure given by a polytope (a consequence of

[1] See also Bensaid et al. (1992) for results on the existence of an interval of prices in the case of transaction costs affecting tradable securities.

[2] See Artzner et al. (1999) for the corresponding result in the study of coherent risk measures and Gilboa and Schmeidler (1989) for the corresponding (dual) representation in the field of choices under uncertainty.

[3] Throughout the paper we assume absence of arbitrage opportunities. Accordingly we avoid using the term "arbitrage-free" or "no-arbitrage" repeatedly.