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The speed of sequential asymptotic learning ☆

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Abstract

In the classical herding literature, agents receive a private signal regarding a binary state of nature, and sequentially choose an action, after observing the actions of their predecessors. When the informativeness of private signals is unbounded, it is known that agents converge to the correct action and correct belief. We study how quickly convergence occurs, and show that it happens more slowly than it does when agents observe signals. However, we also show that the speed of learning from actions can be arbitrarily close to the speed of learning from signals. In particular, the expected time until the agents stop taking the wrong action can be either finite or infinite, depending on the private signal distribution. In the canonical case of Gaussian private signals we calculate the speed of convergence precisely, and show explicitly that, in this case, learning from actions is significantly slower than learning from signals.

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1. Introduction

When making decisions, we often rely on the decisions that others before us have made. Sequential learning models have been used to understand different phenomena that occur when many individuals make decisions based on the observed actions of others. These include herd

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behavior (cf. Banerjee, 1992), where many agents make the same choice, as well as informational cascades (e.g. Bikhchandani et al., 1992), where the actions of the first few agents provide such compelling evidence that later agents no longer have incentive to consider their own private information.

Such results on how information aggregation can fail are complemented by results which demonstrate that when private signals are arbitrarily strong, learning is robust to this kind of collapse (Smith and Sørensen, 2000). In particular, in a process called asymptotic learning (see, e.g., Acemoglu et al., 2011), agents will eventually choose the correct action and their beliefs will converge to the truth. A question that has not been answered in the literature is: how quickly does this happen? And how does the speed of learning compare to a setting in which agents observe signals rather than actions?

We consider the classical setting of a binary state of nature and binary actions, where each of the two actions is optimal at one of the states. The agents receive private signals that are independent conditioned on the state. These signals are unbounded, in the sense that an agent's posterior belief regarding the state can be arbitrarily close to both 0 and 1. The agents are exogenously ordered, and, at each time period, a single agent takes an action, after observing the actions of her predecessors.

We measure the speed of learning by studying how the public belief evolves as more and more agents act. Consider an outside observer who observes the actions of the sequence of agents. The public belief is the posterior belief that such an outside observer assigns to the correct state of nature. It provides a measure of how well the population has learned the state. Since signals are unbounded, the public belief tends to 1 over time (Smith and Sørensen, 2000); equivalently, the corresponding log-likelihood ratio tends to infinity. As the outside observer may also be interested in learning the state, it is natural to ask how quickly she converges to the correct belief, and, in particular, to understand her asymptotic speed of learning when observing actions. Asymptotic rates of convergence are an important tool in the study of inference processes in statistical theory, and have also been studied in social learning models in the Economics literature (e.g., Vives, 1993; Duffie and Manso, 2007; Duffie et al., 2009).

When agents observe the *signals* (rather than actions) of all of their predecessors, this log-likelihood ratio is asymptotically linear. Thus, it cannot grow faster than linearly when the agents observe actions. Our first main finding is that when observing actions, the log-likelihood ratio always grows sub-linearly. Equivalently, the public belief converges sub-exponentially to 1. Our second main finding is that, depending on the choice of private signal distributions, the log-likelihood ratio can grow at a rate that is arbitrarily close to linear.

We next analyze the specific canonical case of Gaussian private signals. Here we calculate precisely the asymptotic behavior of the log-likelihood ratio of the public belief. We show that learning from actions is significantly slower than learning from signals: the log-likelihood ratio behaves asymptotically as $\sqrt{\log t}$. To calculate this we develop a technique that allows, much more generally, for the long-term evolution of the public belief to be calculated for a large class of signal distributions.

Since, in our setting of unbounded signals, agents eventually take the correct action, an additional, natural measure of the speed of learning is the expected time at which this happens: how long does it take until no more mistakes are made? We call this the *time to learn*.

We show that the expected time to learn depends crucially on the signal distributions. For distributions, such as the Gaussian, in which strong signals occur with very small probability, we