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A theory of robust experiments for choice under uncertainty ☆

S. Grant [a,b,*], J. Kline [b], I. Meneghel [a], J. Quiggin [b], R. Tourky [a]

[a] *Australian National University, Australia*
[b] *The University of Queensland, Australia*

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Abstract

Thought experiments are commonly used in the theory of behavior in the presence of risk and uncertainty to test the plausibility of proposed axiomatic postulates. The prototypical examples of the former are the Allais experiments and of the latter are the Ellsberg experiments. Although the lotteries from the former have objectively specified probabilities, the participants in both kinds of experiments may be susceptible to small deviations in their subjective beliefs. These may result from a variety of factors that are difficult to check in an experimental setting: including deviations in the understanding and trust regarding the experiment, its instructions and its method. Intuitively, an experiment is robust if it is tolerant to small deviations in subjective beliefs in models that are in an appropriate way close to the analyst's model. The contribution of this paper lies in the formalization of these ideas.

JEL classification: D81; C91

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* Corresponding author at: Australian National University, Australia.
E-mail address: simon.grant@anu.edu.au (S. Grant).

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1. Introduction

The development of decision theory has been driven, in large measure, by thought experiments questioning the core postulates of the expected utility model, axiomatized for choice under risk by von Neumann and Morgenstern (1944) and for uncertainty by Savage (1954). Experimentalists have gone to great efforts to improve the design of experiments and elicitation of preferences from the participants aimed to test such theories (see Becker et al., 1964; Holt, 1986 and Johnson et al., 2015 amongst others). In this paper we add to these efforts by providing a notion of *robustness* of an experiment that ensures the conclusions of the analyst would not be overturned by introducing a vanishingly small amount of doubt about what was in the minds of the participants. We give a simple way to check for robustness of an experiment and some methods to ensure an experiment is robust.[1]

The main idea is that a robust challenge to a decision-theoretic model arising from an experiment remains a challenge in any model that is close to the analyst's model. With this in mind, we demonstrate that many classic experiments such as the prototypical Allais and Ellsberg experiments and their derivatives (Machina, 2009), although well-conceived, do not pose convincing challenges to the particular decision theory the experiment was designed to test. We show, however, how they can be modified to overcome this problem.

The principal ideas, concepts and results can be readily introduced and illustrated in the context of Ellsberg's single-urn thought experiment. In that thought experiment, the reader is asked to "imagine an urn known to contain 30 red balls and 60 black and yellow balls, the latter in unknown proportion." (Ellsberg, 1961, p. 653). A ball is to be drawn from the urn. On the basis of the color of the ball drawn, first consider a choice between a bet that pays \$100 if the ball drawn is red and nothing otherwise, denoted b_R, and a bet that pays \$100 if the ball drawn is black and nothing otherwise, denoted b_B. Next consider a choice between a bet that pays \$100 if the ball drawn is red or yellow and nothing if it is black, denoted b_{RY}, and a bet that pays \$100 if the ball drawn is black or yellow and nothing if it is red, denoted b_{BY}. Ellsberg argues that anyone exhibiting the preference pattern $b_R \succ b_B$ and $b_{BY} \succ b_{RY}$ is "simply not acting 'as though' they assigned numerical or even qualitative probabilities to the events in question." (Ellsberg, 1961, p. 656). In particular, this means such a preference pattern is inconsistent with subjective expected utility theory.

Ellsberg's reasoning rests on the assumption that the subject in such an experiment takes the state space to be the *sample* space $\{s_R, s_B, s_Y\}$, where s_c is the sample-state in which a ball of color c is drawn from the urn *independent* of which bet has been chosen by the subject in either problem. By identifying each of these three states with the corresponding vector of bet-consequences we obtain the following 4×3 consequence matrix:

$$C = \begin{array}{c} \\ b_R \\ b_B \\ b_{RY} \\ b_{BY} \end{array} \begin{array}{c} \begin{array}{ccc} s_R & s_B & s_Y \end{array} \\ \begin{bmatrix} 100 & 0 & 0 \\ 0 & 100 & 0 \\ 100 & 0 & 100 \\ 0 & 100 & 100 \end{bmatrix} \end{array}.$$

The set of admissible preferences are ones that represent a subjective expected utility maximizing decision-maker characterized by a pair (u, p) where

[1] In fact, we find that some of the methods used by experimentalists have the effect of making the experiments robust in our sense (Halevy, 2007 and Binmore et al., 2012).