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Spectral behavior of pulsed Bessel beams focused by a dispersive lens

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ABSTRACT

Starting from the Huygens–Fresnel diffraction integral, the analytical expression for the power spectrum of pulsed Bessel beams focused by a dispersive aperture lens is derived and used to study the spectral anomalies of pulsed Bessel beams in the focused field. Numerical calculation results are given to illustrate the dependence of spectral anomalous behavior on the pulse parameters, truncation parameter and material dispersion of the lens. It is shown that near the phase singularities the spectral anomalies may take place. The potential applications of spectral anomalies of ultrashort pulsed beams in information encoding and information transmission are considered.

1. Introduction

In recent years studies of the phenomenon associated with phase singularities have gradually developed into a new branch of physical optics, referred as singular optics [1]. More recently, the subject of singular optics has been extended from monochromatic light to polychromatic wave field by Gbur et al. [2–4], in which they have shown that spectral changes occur in the vicinity of intensity zeros in the focal region of polychromatic, spatially coherent, converging spherical waves, that is, the spectrum is redshifted at some points, blueshifted at others, and split into two lines elsewhere. These drastic spectral changes in the vicinity of singular points seem to have a close link to spectral switches, which were found several years ago [5–7], and the link between the spectral anomalies and the spectral switches has been discussed in Ref. [8]. The anomalous spectral behavior of spatially fully and partially coherent steady-state beams have been extensively studied both theoretically and experimentally [9–19], only few publications have dealt with the anomalous spectral behavior of ultrashort pulsed beams [20–23]. This paper is aimed at studying spectral anomalies of diffracted ultrashort pulsed beams focused by a dispersive lens. In Section 2, the pulsed Bessel beam is taken as an example, the expression for the power spectra of pulsed Bessel beams focused by a dispersive lens is derived. Section 3 presents numerical calculation results to illustrate the anomalous behavior of pulsed Bessel beams focused by a dispersive lens and its potential application in information encoding and transmission. In Section 4, a brief summary of the main results concludes this paper.

2. Theoretical model

Consider the optical system shown in Fig. 1, suppose that the spatial–temporal profile of an ultrashort pulsed Bessel beam which is incident upon an aperture lens with full width $2a$ and focal length f at the $z=0$ plane takes the form

$$E(r',0,t)=J_0(\alpha r')A(t) \tag{1}$$

where $J_0(\bullet)$ is the Bessel function of the first kind and order 0, r' denotes the polar radius of point P at the plane $z=0$, α is the spatial constant which is assumed to be independent of frequency ω. $A(t)$ is the temporal profile of the pulsed beam.

Using the Fourier transform, the field in the space–frequency domain is given by

$$E(r',0,\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}E(r',0,t)\exp(\mathrm{i}\omega t)\mathrm{d}t=J_0(\alpha r')A(\omega) \tag{2}$$

where

$$A(\omega)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}f(t)\exp(\mathrm{i}\omega t)\mathrm{d}t \tag{3}$$

is the Fourier spectrum at the source plane $z=0$.

Within the framework of the paraxial approximation, the field of the pulsed Bessel beams focused by a dispersive lens obeys the

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Fig. 1. Schematic illustration of pulsed Bessel beams focused by an aperture lens.

generalized Huygens–Fresnel diffraction integral and is given by [24]

$$E(r,\varphi,z,\omega)=\frac{\mathrm{i}k(\omega)}{2\pi B}\exp\left[-\mathrm{i}k(\omega)z\right]\int_0^a\int_0^{2\pi}E(r',0,\omega)\times\exp\left\{-\frac{\mathrm{i}k(\omega)}{2B}\left[Ar'^2+Dr^2-2r'r\cos(\varphi'-\varphi)\right]\right\}r\,\mathrm{d}r'\,\mathrm{d}\varphi' \tag{4}$$

where r and φ denote polar radius and radius angle of point at the z-plane, k is the wave number related to the wavelength λ by $k=2\pi/\lambda$. The integration is made with respect to the aperture plane.

The *ABCD* matrix reads as

$$\begin{pmatrix}A & B\\ C & D\end{pmatrix}=\begin{pmatrix}-\Delta z & f(\Delta z+1)\\ -1/f & 1\end{pmatrix} \tag{5}$$

where Δz is the relative propagation distance $\Delta z=(z-f)/f$, f denotes the focal length of the lens, which is frequency-dependent and can be expended about the central frequency ω_0 into the series

$$f(\omega)=\sum_{m=0}^{\infty}C_m(\omega-\omega_0)^m=f_0F(\omega) \tag{6}$$

and

$$F(\omega)=1+\xi_1\left(\frac{\omega-\omega_0}{\omega_0}\right)+\xi_2\left(\frac{\omega-\omega_0}{\omega_0}\right)+\ldots \tag{7}$$

$$\xi_1=\frac{\omega_0}{f_0}\frac{\mathrm{d}f}{\mathrm{d}\omega}\bigg|_0 \tag{8}$$

$$\xi_2=\frac{\omega_0^2}{2f_0}\frac{\mathrm{d}^2f}{\mathrm{d}\omega^2}\bigg|_0 \tag{9}$$

where f_0 is the focal length of the lens at the central frequency ω_0, C_m is the coefficient of the series, ξ_1 and ξ_2 are material dispersive parameters of the lens. In the derivation of Eqs. (6)–(9) the following relations have been used:

$$1/f(\omega)=(n(\omega)-1)(1/R_1+1/R_2) \tag{10}$$

$$\frac{\mathrm{d}f(\omega)}{\mathrm{d}\omega}=-\frac{f(\omega)}{n(\omega)-1}\frac{\mathrm{d}n(\omega)}{\mathrm{d}\omega} \tag{11}$$

$$\frac{\mathrm{d}^2f(\omega)}{\mathrm{d}\omega^2}=-\frac{f(\omega)}{n(\omega)-1}\frac{\mathrm{d}^2n(\omega)}{\mathrm{d}\omega^2}+\frac{2f(\omega)}{(n(\omega)-1)^2}\left(\frac{\mathrm{d}n(\omega)}{\mathrm{d}\omega}\right)^2 \tag{12}$$

where $n(\omega)$ is the refraction index of the lens, R_1 and R_2 are curvature radii of front and back surfaces of the lens.

On substituting from Eqs. (2) and (5) into Eq. (4) and performing the integral, the final result can be arranged as

$$E(r,\theta,z,\omega)=\frac{\mathrm{i}z_0}{f_0(\Delta z+1)}\frac{\omega}{\omega_0}\exp\left[-\mathrm{i}k\left(z+\frac{r^2}{2z}\right)\right]A(\omega)\times\int_0^1J_0(a\alpha\rho)J_0\left(\theta\frac{\omega}{\omega_0}\rho\right)\exp\left[-\frac{\mathrm{i}z_0}{2f_0}\left(\frac{1}{\Delta z+1}-\frac{1}{F(\omega)}\right)\frac{\omega}{\omega_0}\rho^2\right]\rho\,\mathrm{d}\rho \tag{13}$$

where

$$z_0=\frac{2\pi a^2}{\lambda_0} \tag{14}$$

$$\theta=\frac{r/a}{z/z_0}\ \text{(normalized diffraction angle)} \tag{15}$$

$$\rho=\frac{r'}{a}\ \text{(relative transversal coordinate)} \tag{16}$$

ω_0 and λ_0 are the central frequency and central wavelength of the original power spectrum, respectively.

Assume that the incident pulse takes a Gaussian form

$$A(t)=\exp\left[-\frac{t^2}{2T^2}\right]\exp(-\mathrm{i}\omega_0t) \tag{17}$$

where T denotes the pulse duration. Substituting Eq. (16) into Eq. (3), straightforward integral calculations deliver

$$A(\omega)=T\exp\left[-\frac{T^2}{2}(\omega-\omega_0)^2\right] \tag{18}$$

Thus, the power spectrum of the incident pulse at the source plane $z=0$ is given by

$$S^{(0)}(\omega)=|A(\omega)|^2=T^2\exp\left[-T^2(\omega-\omega_0)^2\right] \tag{19}$$

From Eqs. (13) and (19) the power spectrum in the focused field turns out to be

$$S(\theta,z,\omega)=|E(r,\theta,z,\omega)|^2=S^{(0)}(\omega)M(\theta,z,\omega) \tag{20}$$

where

$$M(\theta,z,\omega)=\left(\frac{z_0}{f_0(\Delta z+1)}\right)^2\left(\frac{\omega}{\omega_0}\right)^2\left|\int_0^1J_0(a\alpha\rho)J_0\left(\theta\frac{\omega}{\omega_0}\rho\right)\times\exp\left[-\frac{\mathrm{i}z_0}{2f_0}\left(\frac{1}{\Delta z+1}-\frac{1}{F(\omega)}\right)\frac{\omega}{\omega_0}\rho^2\right]\rho\,\mathrm{d}\rho\right|^2 \tag{21}$$

$M(\theta,z,\omega)$ denotes the spectral modifier of focused pulsed Bessel beam. Eqs. (20)–(21) indicate that the power spectrum of focused pulsed Bessel beams is a product of the original spectrum $S^{(0)}(\omega)$ and the spectral modifier $M(\theta,z,\omega)$. The original spectrum $S^{(0)}(\omega)$ is dependent on the pulse duration T. The spectral modifier $M(\theta,z,\omega)$ depends on the aperture parameter a, spatial constant α, dispersive parameters ξ_1, ξ_2, position parameter Δz and diffraction angle θ in general. The power spectrum of pulsed Bessel beams focused by a dispersive-free lens can be treated in a unified way, as a spectral case ($F(\omega)=1$).

Performing the inverse Fourier transform of Eq. (13), the spatiotemporal distribution of pulsed Bessel beams focused by a dispersive lens is expressed as

$$E(\theta,z,t)=\int_{-\infty}^{\infty}E(\theta,z,\omega)\exp(-\mathrm{i}\omega t)\,\mathrm{d}\omega \tag{22}$$

3. Spectral anomalies of diffracted pulsed Bessel beams focused by a dispersive lens at the geometrical focal plane

In the following, numerical calculations were performed with Eqs. (20)–(21), our attention is focused on the spectral anomalies