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Notes

Fairness and group-strategyproofness clash in assignment problems

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Abstract

No group-strategyproof and ex-post Pareto optimal random matching mechanism treats equals equally. Every mechanism that arises out of the randomization over a set of non-bossy and strategyproof mechanisms is non-bossy. Random serial dictatorship, which arises out of a randomization over all deterministic serial dictatorships is non-bossy but not group-strategyproof.

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1. Introduction

An ideal mechanism would be impossible to manipulate, efficient and fair. For house matching problems, where a finite set of agents with linear preferences over houses needs to be matched to these houses, there exists no group-strategyproof and ex-post Pareto optimal random matching mechanism that treats equals equally. So group-strategyproofness, one of the strongest non-

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manipulability requirements, clashes with some of the weakest efficiency- and fairness-criteria: ex-post Pareto optimality and equal treatment of equals. A mechanism that maps every profile of preferences to a lottery over Pareto optima is ex-post Pareto optimal. The mechanism treats equals equally, if any two agents who submit the same preference face the same lottery over houses. It is group-strategyproof if no group ever gains by lying about the group's preferences. Group-strategyproofness strengthens strategyproofness which only requires that no agent ever gains by lying about his preferences.

My result complements Bogomolnaia and Moulin's (2001) theorem that no strategyproof and ordinally efficient matching mechanism treats equals equally. While Bogomolnaia and Moulin and I use the same weak criterion of fairness, our efficiency and non-manipulability requirements differ: where I only impose ex-post Pareto optimality, Bogomolnaia and Moulin require ordinal efficiency; where they only impose strategyproofness, I require group-strategyproofness.

Papai (2000) has given a very useful characterization of group-strategyproofness: a deterministic matching mechanism is group-strategyproof if and only if it is strategyproof and non-bossy, in the sense that no agent can change another agent's outcome without also changing his own. While Barbera, Berga, and Moreno (2014) show that this equivalence extends far beyond house-matching problems, Thompson (2016) provides an overview on where the equivalence holds (and where it fails). I apply Theorem 1 to the question whether this equivalence carries over to random matching mechanisms. Random serial dictatorship, which arises out of a uniform randomization over the order of agents as dictators in serial dictatorship, is ex-post Pareto optimal and treats equals equally. By Theorem 1, random serial dictatorship is not group-strategyproof. To see that Papai's (2000) equivalence result does not extend to the random matching context it only remains to show that random serial dictatorship, which is known to be strategyproof, is non-bossy. To this end, Theorem 2 shows that any randomization over a set of non-bossy and strategyproof mechanisms yields a non-bossy random matching mechanism. Since any serial dictatorship is non-bossy and strategyproof, random serial dictatorship is non-bossy.

The results presented here cover matching problems with and without outside options. In fact, a few minor changes suffice to extend the proofs from one case to the other. This contrasts with other results in matching that crucially depend on the presence (or absence) of outside options. Svensson's (1999) characterization of serial dictatorship does not extend to the case with outside options. Kesten and Kurino (2013) show that while deferred acceptance is an optimal mechanism when considering the full domain of preferences with outside options, improvements upon deferred acceptance are possible if (at least some) agents have no outside options. Erdil (2014) constructs a random matching mechanism that ex-ante Pareto dominates random serial dictatorship when agents have outside options.

2. Definitions

There is a set $N = \{1, \ldots, n\}$ of agents and a set of houses H. The option to stay homeless ($\emptyset$) is always available $\emptyset \in H$. Generic elements of H (including $\emptyset$) are denoted h. A matching is a set of agent-house pairs, denoted as a vector $x \in H^n$ where $x_i = x_j$ and $i \neq j$ imply $x_i = \emptyset$. Under x agent i is unmatched if $x_i = \emptyset$, otherwise house x_i is agent i's is match. The set of all matchings is X. Agent i's preference on H is a linear order $\succsim_i$. So $\succsim_i$ is complete, transitive and $h \sim h'$ implies $h = h'$. A profile of all agents' preferences $(\succsim_i)_{i \in N}$ is denoted $\succsim$, where $\succsim_G$ and $\succsim_{-G}$ are the preferences of all agents in some group $G \subset N$ and outside that group, so $\succsim = (\succsim_G, \succsim_{-G})$. The set Ω is the set of all profiles of linear orders on H. Agents are selfish in the sense that they only consider their own houses when ranking different matchings.